Camp Patch Theory

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1 Overview

Camp is a version control system, similar in philosophy to darcs. In this document we present a description of camp's patch theory.

This document is very much a work in progress. Some sections are more readable than others. Some coq proofs are starting to appear, and where they haven't there may be a (somewhat handwavey) hand proof.

In Section 2 we quickly introduce some general notation that we will be using.

In Section 3 we say define a set of unnamed patches, and explain what behaviour we require that they satisfy. We also introduce a concept of commutation for unnamed patches.

In Section 4 we define sequences of unnamed, and some properties that they must satisfy.

In Section 5 we define a subset of all the possible unnamed patch sequences, which we call sensible.

In Section 6 we introduce names.

Everything up to this point is an input to the paper. In other words, we require that we are provided with unnamed patches, unnamed patch commute and the notion of sensible unnamed patch sequences, which satisfy the properties that we specify. Likewise, we require that we are given a suitable set of names.

While it is possible to start from a lower level, and prove the correctness of the operations on unnamed patches, that is not the goal of this document.

In Section 7 we build named patches out of names and unnamed patches.

In Section **??** we build sequences from these named patches.

In Section 8 we take a break from the formal definition, and give a description of how merges (of non-conflicting changes) work.

In Section 9 we introduce the concept of *patch universes*. This is an algebraic structure that many kinds of patches are examples of.

This actually give you all that you need in order to make a version control system, provided that you never make conflicting changes that you later want to merge.

In Section 10 we introduce a "contexted patch" datastructure.

In Section 12 we define catches and repos, and make conjectures that they satisfy properties that we want them to satisfy.

In Section 13 we prove that catches form a patch universe.

The result is a version control system in which it is always possible to merge repos — which is vital for a distributed version control system.

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printing $<^\sim>\mathrm{u} \leftrightsquigarrow_\mathrm{u}$ printing $<^\sim ?^\sim>\mathrm{u} \overset{?}{\leftrightsquigarrow}_\mathrm{u}$ printing $\square\mathrm{u}\ \epsilon_\mathrm{u}$

printing $<^\sim> \leftrightsquigarrow$ printing $<^\sim ?^\sim> \overset{?}{\leftrightsquigarrow}$ printing $\square\ \epsilon$ notation

2 Notation

We write $\forall a \in A, b \in B \cdot p\ a\ b$ to mean "for all a in A and b in B, $p\ a\ b$ holds".

We will be defining some relations which relate pairs and singles of values. In order to be clear about what the relation is acting on, we will write the single value v as $\langle v \rangle$, and the pair of values v and w as $\langle v, w \rangle$.

There are certain letters that we use to represent certain types of thing. These things may not be familiar to you yet, but you can refer back to this section as you need to later on. They are:

Unnamed Patches	$\underline{p}$, $\underline{q}$, $\underline{r}$, $\underline{s}$, $\underline{t}$, $\underline{u}$, $\underline{v}$
(Named) Patches	p, q, r, s, t, u, v
Contexted patches	w, x, y, z
Catches	c, d, e, f, g

If we are using k to represent a thing, then we will use $\overline{k}$ to represent a sequence of things, and K to represent a set of things.

We use _ to mean "some value that we do not wish to name". Multiple _s do not necessarily refer to the same value.

We use subsections of *italic text* when giving intuition about what the formal description means.

The coq code is typeset thus:

```
(*  Here comes the science bit. Concentrate! *)
```

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printing $<\tilde{}> \leftrightsquigarrow$ printing $<\tilde{}?\tilde{}> \overset{?}{\leftrightsquigarrow}$ printing $\square\ \epsilon$ unnamed_patches

3 Unnamed Patches

```
Module Export UNNAMED_PATCHES.
```

We start by introducing *unnamed patches*, and the concept of *patch commutation*. For now, we will refer to unnamed patches as simply *patches*.

Definition 3.1 (unnamed-patches)
We have a (possibly infinite) set of patches $\underline{\mathbf{P}}$.

```
Reserved Notation "p ^u" (at level 10).
Reserved Notation "« p , q » <~>u « q' , p' »" (at level 60, no associativity).
Reserved Notation "p <~?~>u q" (at level 60, no associativity).

Record UnnamedPatch : Type := mkUnnamedPatch {
    up_type : Set;
```

The underlining in the $\underline{\mathbf{P}}$ and $\underline{p}$ syntax is a bit heavy, but we prefer to reserve the cleaner syntax for named patches.

Axiom 3.1 (unnamed-patch-inverse)
$\forall \underline{p} \in \underline{\mathbf{P}} \cdot \underline{p}^{-1} \in \underline{\mathbf{P}}$.

```
(*
XXX inverse is an axiom in the latex. Maybe it should
be a definition?
*)
```

```
unnamedPatchInverse : up_type → up_type
    where "p ^u" := (unnamedPatchInverse p);
```

Explanation
All patches have an inverse.

```
unnamedPatchInvertInverse : ∀ (p : up_type), (p^u)^u = p;
```

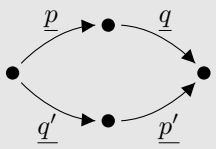
Definition 3.2 (unnamed-patch-commute)
There is a $\leftrightarrow$ relation, pronounced "commutes to", such that:
$\forall \underline{p} \in \mathbf{P}, \underline{q} \in \mathbf{P} \cdot$
$\quad (\exists \underline{p'} \in \mathbf{P}, \underline{q'} \in \mathbf{P} \cdot \langle \underline{p}, \underline{q} \rangle \leftrightarrow \langle \underline{q'}, \underline{p'} \rangle)$
$\vee \langle \underline{p}, \underline{q} \rangle \leftrightarrow \text{fail}$

```
unnamedPatchCommute : up_type → up_type → up_type → up_type → Prop
    where "« p , q » <~>u « q' , p' »" := (unnamedPatchCommute p q q' p');
```

Explanation
We write $\langle \underline{p}, \underline{q} \rangle \leftrightarrow \langle \underline{q'}, \underline{p'} \rangle$ if $\underline{p}$ and $\underline{q}$ commute, resulting in $\underline{q'}$ and $\underline{p'}$.In other words, doing $\underline{p}$ and then $\underline{q}$ is equivalent to doing $\underline{q'}$ and then $\underline{p'}$, where $\underline{p}$ and $\underline{p'}$ are morally equivalent, and likewise $\underline{q}$ and $\underline{q'}$ are morally equivalent.

For example, adding "Hello" to line 3 of a file and then adding "World" to line 5 of the file is equivalent to adding "World" to line 4 of the file and then adding "Hello" to line 3 of a file. We therefore say that $\langle$add "Hello" 3, add "World" 5$\rangle \leftrightarrow \langle$add "World" 4, add "Hello" 3$\rangle$.

We can represent this diagramatically thus:



Here the dots are repository states, and the arrows are patches; $\underline{pq}$ and $\underline{q'p'}$ get us from the same state, to the same state, but via different intermediate states.

We write $\langle \underline{p}, \underline{q} \rangle \leftrightarrow$ fail if $\underline{p}$ and $\underline{q}$ do not commute. In other words, it is not possible to do the moral equivalent of $\underline{q}$ before $\underline{p}$.

For example, $\langle$replace line 3 with "Hello", replace line 3 with "World"$\rangle \leftrightarrow$ fail

Axiom 3.2 (unnamed-patch-commute-unique)
$\forall \underline{p} \in \mathbf{P}, \underline{q} \in \mathbf{P}, j \in (\mathbf{P} \times \mathbf{P}) \cup \{\text{fail}\}, k \in (\mathbf{P} \times \mathbf{P}) \cup \{\text{fail}\} \cdot$
$(\langle \underline{p}, \underline{q} \rangle \leftrightarrow j) \wedge (\langle \underline{p}, \underline{q} \rangle \leftrightarrow k) \Rightarrow j = k$

```
UnnamedPatchCommuteUnique :
        ∀ (p : up_type) (q : up_type)
                (p' : up_type) (q' : up_type)
```

```
                (p'' : up_type) (q'' : up_type),
    «p , q» <~>u «q' , p'»
  ∧ «p , q» <~>u «q'', p''»
  → (p' = p'') ∧ (q' = q'');
```

Explanation
Either $\underline{p}$ *and* $\underline{q}$ always *commute, or they* never *commute. If they do commute, then the result is always the same.*

Now that we know that the result of a commute is unique, we can afford to be a bit lax about quantifying over variables. For example, if we are dealing with two patches $\underline{p}$ and $\underline{q}$ and we say that $\langle \underline{p}, \underline{q} \rangle \leftrightarrow \langle \underline{q}', \underline{p}' \rangle$ then it is implied that we are considering the case where $\underline{p}$ and $\underline{q}$ commute, and that the result of the commutation is $\underline{q}'$ and $\underline{p}'$.

Definition 3.3 (unnamed-patches-commutable)
We define $\underset{?}{\leftrightarrow}$ to relate two patches if they are commutable, i.e. $\underline{p} \underset{?}{\leftrightarrow} \underline{q} \Leftrightarrow \exists \underline{p}', \underline{q}' \cdot \langle \underline{p}, \underline{q} \rangle \leftrightarrow \langle \underline{q}', \underline{p}' \rangle$

```
unnamedPatchCommutable : up_type → up_type → Prop
 := fun (p : up_type) ⇒ fun (q : up_type) ⇒
    (∃ p' : up_type, ∃ q' : up_type,
    «p , q» <~>u «q' , p'»)
where "p <~?~>u q" := (unnamedPatchCommutable p q);

unnamedPatchCommutable_dec :
      ∀ (p : up_type) (q : up_type),
      {p <~?~>u q} + {~(p <~?~>u q)};
```

Axiom 3.3 (unnamed-patch-commute-self-inverse)
$\forall \underline{p} \in \mathbf{P}, \underline{q} \in \mathbf{P}, \underline{p}' \in \mathbf{P}, \underline{q}' \in \mathbf{P} \cdot$
$(\langle \underline{p}, \underline{q} \rangle \leftrightarrow \langle \underline{q}', \underline{p}' \rangle) \Leftrightarrow (\langle \underline{q}', \underline{p}' \rangle \leftrightarrow \langle \underline{p}, \underline{q} \rangle)$

```
UnnamedPatchCommuteSelfInverse :
      ∀ (p : up_type) (q : up_type)
              (p' : up_type) (q' : up_type),
      («p , q» <~>u «q' , p'») ↔
      («q' , p'» <~>u «p , q»);
```

Explanation
If you commute a pair of patches, and then commute them back again, then you end up back where you started. Note also that we require that if commuting one way succeeds then commuting the other way also succeeds.

Axiom 3.4 (unnamed-patch-commute-square)
$\forall \underline{p} \in \mathbf{P}, \underline{q} \in \mathbf{P}, \forall \underline{p}' \in \mathbf{P}, \underline{q}' \in \mathbf{P} \cdot$
$(\langle \underline{p}, \underline{q} \rangle \leftrightarrow \langle \underline{q}', \underline{p}' \rangle) \Leftrightarrow (\langle \underline{q}'^{-1}, \underline{p} \rangle \leftrightarrow \langle \underline{p}', \underline{q}^{-1} \rangle)$

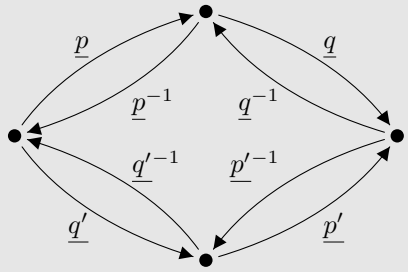
```
UnnamedPatchCommuteSquare :
      ∀ (p : up_type) (q : up_type)
             (p' : up_type) (q' : up_type),
      «p , q» <~>u «q' , p'» →
      « q'^u , p» <~>u «p' , q^u»;

(*  XXX UnnamedPatchCommuteConsistent12 copy/pasted from
       unnamed_patch_sequences without accompanying text *)
UnnamedPatchCommuteConsistent1 :
      ∀ {p1 : up_type}
             {q1 : up_type}
             {r1 : up_type}
             {q2 : up_type}
             {r2 : up_type}
             {q3 : up_type}
             {r3 : up_type}
             {p3 : up_type}
             {p5 : up_type},
      «q1 , r1» <~>u «r2 , q2»
   → «p1 , q1» <~>u «q3 , p5»
   → «p5 , r1» <~>u «r3 , p3»
   → ∃ r4 : up_type,
      ∃ q4 : up_type,
      ∃ p6 : up_type,
      «q3 , r3» <~>u «r4 , q4» ∧
      «p1 , r2» <~>u «r4 , p6» ∧
      «p6 , q2» <~>u «q4 , p3»;

UnnamedPatchCommuteConsistent2 :
      ∀ {p3 : up_type}
             {q3 : up_type}
             {r3 : up_type}
             {q4 : up_type}
             {r4 : up_type}
             {q1 : up_type}
             {r1 : up_type}
             {p1 : up_type}
             {p5 : up_type},
      «q3 , r3» <~>u «r4 , q4»
   → «r3 , p3» <~>u «p5 , r1»
   → «q3 , p5» <~>u «p1 , q1»
   → ∃ r2 : up_type,
      ∃ q2 : up_type,
      ∃ p6 : up_type,
      «q1 , r1» <~>u «r2 , q2» ∧
      «q4 , p3» <~>u «p6 , q2» ∧
      «r4 , p6» <~>u «p1 , r2»
}.
```

Explanation

Axiom 3.1 tells us that all patches have an inverse. Therefore we can augment our patch commutation diagram with inverse patches:



This axiom tells us that we can rotate the diagram 90 degrees clockwise and we again have a valid commutation diagram. The arrows from left to right along the top are $\underline{q'}^{-1}\underline{p}$ *and along the bottom are* $\underline{p'}\underline{q}^{-1}$*, thus* $\langle \underline{q'}^{-1}, \underline{p} \rangle \hookleftarrow \langle \underline{p'}, \underline{q}^{-1} \rangle$.

In actual fact, by applying this axiom twice or three times, we can see that all four of the rotations are equivalent, and thus $\langle \underline{p'}^{-1}, \underline{q'}^{-1} \rangle \hookleftarrow \langle \underline{q}^{-1}, \underline{p}^{-1} \rangle$ *and* $\langle \underline{q}, \underline{p'}^{-1} \rangle \hookleftarrow \langle \underline{p}^{-1}, \underline{q'} \rangle$.

```
Notation "p ^u" := (unnamedPatchInverse _ p).
Notation "« p , q » <˜>u « q' , p' »" := (unnamedPatchCommute _ p q q' p').
Notation "p <˜?˜>u q" := (unnamedPatchCommutable _ p q).
End UNNAMED_PATCHES.
```

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printing $<\tilde{}>$ $\leftrightsquigarrow$ printing $<\tilde{}?\tilde{}>$ $\overset{?}{\leftrightsquigarrow}$ printing $\square$ ϵ unnamed_patch_sequences

4 Unnamed Patch Sequences

```
Module Export patch_sequences.
Require Import unnamed_patches.
```

Definition 4.1 (unnamed-patch-sequences)
We write the empty sequence of unnamed patches as ϵ.

We compose unnamed patch sequences with juxtaposition.

Definition 4.2 (unnamed-patch-sequence-commute)
We extend $\hookleftarrow$ to work with combinations of patches and patch sequences in the natural way:

Commuting a patch with a sequence:
$\langle \underline{p}, \epsilon \rangle \hookleftarrow \langle \epsilon, \underline{p} \rangle$
$\langle \underline{p}, \underline{q}\overline{r} \rangle \hookleftarrow \left\langle \underline{q'}\overline{r'}, \underline{p'} \right\rangle$
if $\langle \underline{p}, \underline{q} \rangle \hookleftarrow \langle \underline{q'}, \underline{p''} \rangle$
$\langle \underline{p''}, \overline{r} \rangle \hookleftarrow \left\langle \overline{r'}, \underline{p'} \right\rangle$

Commuting a sequence with a patch:

$\langle \epsilon, \underline{p} \rangle \leftrightarrow \langle \underline{p}, \epsilon \rangle$
$\langle \overline{\underline{p}} q, \underline{r} \rangle \leftrightarrow \left\langle \underline{r'}, \overline{\underline{p}'} q' \right\rangle$
if $\langle \underline{q}, \underline{r} \rangle \leftrightarrow \langle \underline{r''}, \underline{q'} \rangle$
$\langle \overline{\underline{p}}, \underline{r''} \rangle \leftrightarrow \left\langle \underline{r'}, \overline{\underline{p}'} \right\rangle$

Commuting a sequence with another sequence:
$\left\langle \overline{\underline{p}}, \epsilon \right\rangle \leftrightarrow \left\langle \epsilon, \overline{\underline{p}} \right\rangle$
$\langle \epsilon, \underline{p} \rangle \leftrightarrow \langle \underline{p}, \epsilon \rangle$
$\langle \overline{\underline{p}} q, \underline{r} \overline{s} \rangle \leftrightarrow \left\langle \underline{r'' \overline{s}''}, \overline{\underline{p}''} q'' \right\rangle$
if $\langle \underline{q}, \underline{r} \rangle \leftrightarrow \langle \underline{r'}, \underline{q'} \rangle$
$\langle \overline{\underline{p}}, \underline{r'} \rangle \leftrightarrow \left\langle \underline{r''}, \overline{\underline{p}'} \right\rangle$
$\langle \underline{q'}, \overline{\underline{s}} \rangle \leftrightarrow \left\langle \overline{\underline{s}'}, \underline{q''} \right\rangle$
$\left\langle \overline{\underline{p}'}, \overline{\underline{s}'} \right\rangle \leftrightarrow \left\langle \overline{\underline{s}''}, \overline{\underline{p}''} \right\rangle$

Note that the first two definitions are degenerate cases of the third.

In all cases, if the above rules don't apply the commute fails.

Axiom 4.1 (unnamed-patch-commute-preserves-commute)
$\forall \underline{p} \in \mathbf{P}, \underline{q} \in \mathbf{P}, \underline{r} \in \mathbf{P}, \underline{p'} \in \mathbf{P}, \underline{q'} \in \mathbf{P}, \underline{r'} \in \mathbf{P} \cdot$
$\left(\langle \underline{p}, \underline{qr} \rangle \leftrightarrow \langle \underline{q'r'}, \underline{p'} \rangle\right) \Rightarrow \left(\left(\underline{q} \underset{?}{\leftrightarrow} \underline{r}\right) \Leftrightarrow \left(\underline{q'} \underset{?}{\leftrightarrow} \underline{r'}\right)\right)$

Explanation
This axiom says that if two patches commute then they still commute if you commute something else past both of them.

Likewise, if two patches do not commute, then they still do not commute after commuting something else past them.

Axiom 4.2 (unnamed-patch-commute-consistent)
$\forall \underline{p} \in \mathbf{P}, \underline{q} \in \mathbf{P}, \underline{r} \in \mathbf{P}, \underline{p'} \in \mathbf{P}, \underline{q'} \in \mathbf{P}, \underline{r'} \in \mathbf{P} \cdot$
$\left(\langle \underline{q}, \underline{r} \rangle \leftrightarrow \langle \underline{r'}, \underline{q'} \rangle\right) \Rightarrow \left(\left(\langle \underline{p}, \underline{qr} \rangle \leftrightarrow \langle \_, \underline{p'} \rangle\right) \Leftrightarrow \left(\langle \underline{p}, \underline{r'q'} \rangle \leftrightarrow \langle \_, \underline{p'} \rangle\right)\right)$

```
Axiom UnnamedPatchCommuteConsistent1 :
      ∀ {unnamedPatch : UnnamedPatch}
              {p1 : up_type unnamedPatch}
              {q1 : up_type unnamedPatch}
              {r1 : up_type unnamedPatch}
              {q2 : up_type unnamedPatch}
              {r2 : up_type unnamedPatch}
              {q3 : up_type unnamedPatch}
              {r3 : up_type unnamedPatch}
              {p3 : up_type unnamedPatch}
              {p5 : up_type unnamedPatch},
      «q1, r1» <~>u «r2, q2»
   → «p1, q1» <~>u «q3, p5»
   → «p5, r1» <~>u «r3, p3»
   → (∃ r4 : up_type unnamedPatch,
       (∃ q4 : up_type unnamedPatch,
        (∃ p6 : up_type unnamedPatch,
          «q3, r3» <~>u «r4, q4» ∧
          «p1, r2» <~>u «r4, p6» ∧
```

```
        «p6, q2» <˜>u «q4, p3»))).
Axiom UnnamedPatchCommuteConsistent2 :
      ∀ {unnamedPatch : UnnamedPatch}
              {p3 : up_type unnamedPatch}
              {q3 : up_type unnamedPatch}
              {r3 : up_type unnamedPatch}
              {q4 : up_type unnamedPatch}
              {r4 : up_type unnamedPatch}
              {q1 : up_type unnamedPatch}
              {r1 : up_type unnamedPatch}
              {p1 : up_type unnamedPatch}
              {p5 : up_type unnamedPatch},
      «q3, r3» <˜>u «r4, q4»
   → «r3, p3» <˜>u «p5, r1»
   → «q3, p5» <˜>u «p1, q1»
   → (∃ r2 : up_type unnamedPatch,
       (∃ q2 : up_type unnamedPatch,
         (∃ p6 : up_type unnamedPatch,
           «q1, r1» <˜>u «r2, q2» ∧
           «q4, p3» <˜>u «p6, q2» ∧
           «r4, p6» <˜>u «p1, r2»))).
```

Explanation
This axiom says that commuting a patch p past two patches qr gives you the same patch p' regardless of whether q and r have been commuted or not.

```
End patch_sequences.
```

5 Sensible Unnamed Patch Sequences

Definition 5.1 (sensible-sequences)
We classify some unnamed patch sequences as *sensible*.

Explanation
For example, the sequence 'add file foo; insert "hello world" into foo' is sensible, whereas 'insert "hello world" into foo' is not sensible, as it doesn't make sense to insert text into a file without first adding the file.

Axiom 5.1 (doing-nothing-is-sensible)
ϵ is sensible.

Explanation
If you haven't done anything then you certainly haven't done anything that isn't sensible.

Axiom 5.2 (sensible-subsequences)
If $\underline{\overline{p}q}$ is sensible then $\underline{\overline{p}}$ is sensible.

Explanation
If you are doing sensible things, then you can stop at any time, and the rseult will be sensible.

Axiom 5.3 (commute-preserves-sensibility)
If $\underline{\overline{p}qr\overline{s}}$ is sensible and $\langle \underline{q}, \underline{r} \rangle \leftrightarrow \langle \underline{r'}, \underline{q'} \rangle$, then $\underline{\overline{p}r'q'\overline{s}}$ is sensible.

Explanation
Commuting patches maintains sensibility.

Axiom 5.4 (sensible-inverse)
If $\underline{\overline{pq}}$ is sensible then $\underline{\overline{pqq}}^{-1}$ is sensible.

Explanation
Undoing patches takes us back to an earlier sensible situation.

coqdoc

printing <~>u $\leftrightsquigarrow_{\mathrm{u}}$ printing <~?~>u $\overset{?}{\leftrightsquigarrow}_{\mathrm{u}}$ printing □u ϵ_{u}

printing <~> $\leftrightsquigarrow$ printing <~?~> $\overset{?}{\leftrightsquigarrow}$ printing □ ϵ names

6 Names

```
Module Export NAMES.

Require Import Coq.MSets.MSets.
Require Import Coq.Structures.Orders.
Require Import Coq.Structures.OrdersAlt.
```

Definition 6.1 (names)
We have a (possibly infinite) set of names **N**.

```
Parameter Name : Set.
Parameter Name_compare : Name → Name → comparison.
Parameter Name_compare_sym : ∀ {x y : Name},
                               Name_compare y x = CompOpp (Name_compare x y).
Parameter Name_compare_trans : ∀ {c : comparison}
                                 {x y z : Name},
                                 Name_compare x y = c
                               → Name_compare y z = c
                               → Name_compare x z = c.
Parameter Name_eq_leibniz : ∀ {s s' : Name},
                              Name_compare s s' = Eq
                            → s = s'.
```

Definition 6.2 (names-signed)
We say that a name is either positive or negative.

```
Inductive Sign : Set
  := Positive | Negative.

Inductive SignedName : Set
  := MkSignedName : ∀ (s : Sign)
                      (n : Name),
                    SignedName.

Definition SignedName_compare (x y : SignedName) : comparison
```

```
    := match x, y with
        | MkSignedName Negative _, MkSignedName Positive _ ⇒ Lt
        | MkSignedName Positive _, MkSignedName Negative _ ⇒ Gt
        | MkSignedName Negative x', MkSignedName Negative y' ⇒ Name_compare y' x'
        | MkSignedName Positive x', MkSignedName Positive y' ⇒ Name_compare x' y'
        end.

Lemma SignedName_compare_sym : ∀ {x y : SignedName},
                               SignedName_compare y x = CompOpp
(SignedName_compare x y).
Proof with auto.
intros.
destruct x as [sx nx].
destruct y as [sy ny].
destruct sx; destruct sy;
unfold SignedName_compare; auto; apply Name_compare_sym.
Qed.

Lemma SignedName_compare_trans :
    ∀ c x y z,
    (SignedName_compare x y) = c
 → (SignedName_compare y z) = c
 → (SignedName_compare x z) = c.
Proof with auto.
intros.
destruct x as [sx nx].
destruct y as [sy ny].
destruct z as [sz nz].
destruct sx; destruct sy; destruct sz;
unfold SignedName_compare in ×...
            apply (Name_compare_trans H H0).
        congruence.
    congruence.
apply (Name_compare_trans H0 H).
Qed.

Lemma SignedName_eq_leibniz :
    ∀ {x y : SignedName},
    SignedName_compare x y = Eq → x = y.
Proof with auto.
intros.
destruct x as [sx nx].
destruct y as [sy ny].
destruct sx; destruct sy;
unfold SignedName_compare in ×.
            apply Name_eq_leibniz in H.
            subst...
        congruence.
    congruence.
apply Name_eq_leibniz in H.
subst...
Qed.
```

Explanation
Here's some intuition for what this means: When you record a patch in camp, the patch name is positive (think: it adds something to the repo). If you roll back that patch, then camp makes the corresponding negative patch (which removes something from the repo).

Axiom 6.1 (names-invertible)
$\forall j \in \mathbf{N} \cdot j^{-1} \in \mathbf{N}$.

```
Definition signedNameInverse (n : SignedName) : SignedName
    := match n with
       | MkSignedName Positive b ⇒ MkSignedName Negative b
       | MkSignedName Negative b ⇒ MkSignedName Positive b
       end.

Lemma nameInverseInverse : ∀ (sn : SignedName), signedNameInverse (signedNameInverse sn) =
sn.
Proof with auto.
intros.
destruct sn.
destruct s...
Qed.

Lemma namesInverseInjective: ∀ (sn1: SignedName) (sn2: SignedName),
                                signedNameInverse sn1 = signedNameInverse sn2 → sn1 = sn2.
Proof with auto.
intros.
destruct sn1;
destruct s;
destruct sn2;
destruct s;
unfold signedNameInverse in *;
inversion H...
Qed.
```

Explanation
Names have an inverse.

Axiom 6.2 (name-inverse-sign)
If $j \in \mathbf{N}$ is positive then j^{-1} is negative. If $j \in \mathbf{N}$ is negative then j^{-1} is positive.

```
(*  XXX TODO *)
```

Explanation
The inverse of a normal patch is a rollback, and vice-versa.

```
Module NameOrderedTypeAlt.
Definition t := Name.
Definition compare := Name_compare.
Definition compare_sym := @Name_compare_sym.
Definition compare_trans := @Name_compare_trans.
End NameOrderedTypeAlt.

Module NameOrderedType := OT_from_Alt(NameOrderedTypeAlt).

Module NameOrderedTypeWithLeibniz.
```

```
Include NameOrderedType.
Definition eq_leibniz := @Name_eq_leibniz.
End NameOrderedTypeWithLeibniz.

Module NameSetMod := MSetList.MakeWithLeibniz(NameOrderedTypeWithLeibniz).

Notation NameSet := (NameSetMod.t).
Notation NameSetIn := (NameSetMod.In).
Notation NameSetAdd := (NameSetMod.add).
Notation NameSetRemove := (NameSetMod.remove).
Notation NameSetEqual := (NameSetMod.Equal).

Module NameSetDec := WDecide (NameSetMod).
Ltac nameSetDec := NameSetDec.fsetdec.

Definition NameSetEquality {s s' : NameSet}
                           (H : NameSetEqual s s')
                         : s = s'
    := NameSetMod.eq_leibniz H.

Lemma NameSet_eq_dec : ∀ (x y : NameSet),
                       {x = y} + {x ≠ y}.
Proof with auto.
intros.
destruct (NameSetMod.eq_dec x y).
    left.
    apply NameSetEquality...
right.
intro.
subst.
elim n.
reflexivity.
Qed.

Module SignedNameOrderedTypeAlt.
Definition t := SignedName.
Definition compare := SignedName_compare.
Definition compare_sym := @SignedName_compare_sym.
Definition compare_trans := @SignedName_compare_trans.
End SignedNameOrderedTypeAlt.

Module SignedNameOrderedType := OT_from_Alt(SignedNameOrderedTypeAlt).

Module SignedNameOrderedTypeWithLeibniz.
Include SignedNameOrderedType.
Definition eq_leibniz := @SignedName_eq_leibniz.
End SignedNameOrderedTypeWithLeibniz.

Module SignedNameSetMod := MSetList.MakeWithLeibniz(SignedNameOrderedTypeWithLeibniz).

Notation SignedNameSet := (SignedNameSetMod.t).
Notation SignedNameSetIn := (SignedNameSetMod.In).
Notation SignedNameSetAdd := (SignedNameSetMod.add).
Notation SignedNameSetRemove := (SignedNameSetMod.remove).
Notation SignedNameSetEqual := (SignedNameSetMod.Equal).

Module SignedNameSetDec := WDecide (SignedNameSetMod).
Ltac signedNameSetDec := SignedNameSetDec.fsetdec.

Definition SignedNameSetEquality {s s' : SignedNameSet}
```

```
                                        (H : SignedNameSetEqual s s')
                                      : s = s'
    := SignedNameSetMod.eq_leibniz H.

Lemma SignedNameSet_eq_dec : ∀ (x y : SignedNameSet),
                                     {x = y} + {x ≠ y}.
Proof with auto.
intros.
destruct (SignedNameSetMod.eq_dec x y).
    left.
    apply SignedNameSetEquality...
right.
intro.
subst.
elim n.
reflexivity.
Qed.

End NAMES.
```

coqdoc

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printing <˜> $\rightsquigarrow$ printing <˜?˜> $\overset{?}{\rightsquigarrow}$ printing □ ϵ named_patches

7 Named patches

```
Module Export NAMED_PATCHES.

Require Import Equality.
Require Import ProofIrrelevance.

Require Import unnamed_patches.
Require Import names.
Require Import patch_universes.
Require Import invertible_patchlike.
```

Rather than working directly with unnamed patches, we will work with named patches. A named patch is built out of a name and an unnamed patch. For the motivation for this decision, see Appendix B.

Definition 7.1 (named-patches)
If $j \in \mathbf{N}$ and $\underline{p} \in \underline{\mathbf{P}}$, then $[\![j, p]\!]$ is a *named patch*.

We define a (possibly infinite) set of *named patches* $\mathbf{P}$ thus: $\forall j \in \mathbf{N}, \underline{p} \in \underline{\mathbf{P}} \cdot [\![j, p]\!] \in \mathbf{P}$.

```
Record NamedPatch (unnamedPatch : UnnamedPatch)
                  (from to : NameSet) : Type := mkNamedPatch {
    np_unnamedPatch : up_type unnamedPatch;
    np_name : SignedName;
    np_nameOK : patchNamesOK from to np_name
```

```
}.
Implicit Arguments np_unnamedPatch [unnamedPatch from to].
Implicit Arguments np_name [unnamedPatch from to].
Implicit Arguments np_nameOK [unnamedPatch from to].
Implicit Arguments mkNamedPatch [unnamedPatch from to].
```

From here on, the unqualified term "patches" will refer to named patches.

Definition 7.2 (patch-name)
We define n to tell us the name of a patch, i.e. $n\left(\left[\!\left[j, \underline{p}\right]\!\right]\right) = j$.

Definition 7.3 (unnamed-patch-of)
We define $\lfloor p \rfloor$ to tell us the unnamed patch of a patch, i.e. $\left\lfloor \left[\!\left[j, \underline{q}\right]\!\right] \right\rfloor = \underline{q}$.

Definition 7.4 (named-patch-inverse)
$\left[\!\left[j, \underline{p}\right]\!\right]^{-1} = \left[\!\left[j^{-1}, \underline{p}^{-1}\right]\!\right]$

Explanation
The inverse of a named patch has the inverse name, and the inverse effect.

```
(*  XXX Should this be elsewhere?: *)
Lemma patchNamesOKInverse : ∀ {from to : NameSet}
                              {n : SignedName}
                              (namesOK : patchNamesOK from to n),
                            patchNamesOK to from (signedNameInverse n).
Proof with auto.
intros.
destruct n.
destruct s.
    simpl.
    destruct namesOK.
    split...
simpl.
destruct namesOK.
split...
Qed.

Definition namedPatchInverse {unnamedPatch : UnnamedPatch}
                             {from to : NameSet}
                             (p : NamedPatch unnamedPatch from to)
                           : NamedPatch unnamedPatch to from
(*  XXX Shouldn't unnamedPatchInverse's first argument be implicit? *)
 := mkNamedPatch (unnamedPatchInverse _ (np_unnamedPatch p))
                 (signedNameInverse (np_name p))
                 (patchNamesOKInverse (np_nameOK p)).
Notation "p ^" := (namedPatchInverse p)
    (at level 10).
```

Definition 7.5 (named-patch-commute)
We extend $\leftrightarrow$ to named patches thus:
$\left(\left\langle \left[\!\left[j, \underline{p}\right]\!\right], \left[\!\left[k, \underline{q}\right]\!\right] \right\rangle \leftrightarrow \left\langle \left[\!\left[k, \underline{q'}\right]\!\right], \left[\!\left[j, \underline{p'}\right]\!\right] \right\rangle\right) \Leftrightarrow (j \neq k) \wedge \left(j \neq k^{-1}\right) \wedge \left(\langle p, q \rangle \leftrightarrow \langle q', p' \rangle\right)$

```
Inductive namedPatchCommute {unnamedPatch : UnnamedPatch}
                            {from mid1 mid2 to : NameSet}
          : NamedPatch unnamedPatch from mid1
         → NamedPatch unnamedPatch mid1 to
         → NamedPatch unnamedPatch from mid2
         → NamedPatch unnamedPatch mid2 to
         → Prop
     := MkNamedPatchCommute : ∀ (up : up_type unnamedPatch) (uq : up_type unnamed-
Patch)
                                (up' : up_type unnamedPatch) (uq' : up_type un-
namedPatch)
                                (np : SignedName) (nq : SignedName)
                                (not_equal : np ≠ nq)
                                (not_inverse : np ≠ signedNameInverse nq)
                                (pOK : patchNamesOK from mid1 np)
                                (qOK : patchNamesOK mid1 to nq)
                                (q'OK : patchNamesOK from mid2 nq)
                                (p'OK : patchNamesOK mid2 to np),
              unnamedPatchCommute unnamedPatch up uq uq' up'
            → namedPatchCommute (mkNamedPatch up np pOK)
                                (mkNamedPatch uq nq qOK)
                                (mkNamedPatch uq' nq q'OK)
                                (mkNamedPatch up' np p'OK).
Notation "« p , q » <~> « q' , p' »"
     := (namedPatchCommute p q q' p')
     (at level 60, no associativity).
```

Explanation

Named patches do not commute with themself or their inverse. Other than that, they commute iff their unnamed patches commute.

Lemma 7.1 (named-patch-commute-unique)
$\forall p \in \mathbf{P}, q \in \mathbf{P}, j \in (\mathbf{P} \times \mathbf{P}) \cup \{\text{fail}\}, k \in (\mathbf{P} \times \mathbf{P}) \cup \{\text{fail}\} \cdot$
$(\langle p, q\rangle \leftrightarrow j) \wedge (\langle p, q\rangle \leftrightarrow k) \Rightarrow j = k$

```
(*  We have to split this into 2 parts in the coq, as we can't
   directly say that q' = q'' as they have different types. *)
Lemma NamedPatchCommuteUnique1 :
      ∀ {unnamedPatch : UnnamedPatch}
            {from mid mid' mid'' to : NameSet}
            {p : NamedPatch unnamedPatch from mid} {q : NamedPatch unnamedPatch
mid to}
            {q' : NamedPatch unnamedPatch from mid'} {p' : NamedPatch unnamed-
Patch mid' to}
            {q'' : NamedPatch unnamedPatch from mid''} {p'' : NamedPatch unnamed-
Patch mid'' to},
      «p, q» <~> «q', p'»
   → «p, q» <~> «q'', p''»
   → (mid' = mid'').
Proof with auto.
intros unnamedPatch from mid mid' mid'' to p q q' p' q'' p'' commute' commute''.
dependent destruction commute'.
```

```
dependent destruction commute".
assert (NameSetEqual mid' mid").
    clear - q'OK q'OK0.
    destruct nq.
    destruct s.
        unfold patchNamesOK in ×.
        inversion_clear q'OK.
        inversion_clear q'OK0.
        nameSetDec.
    unfold patchNamesOK in ×.
    admit.
    (*  nameSetDec. *)
apply NameSetEquality...
Qed.

Lemma NamedPatchCommuteUnique2 :
      ∀ {unnamedPatch : UnnamedPatch}
              {from mid mid' to : NameSet}
              {p : NamedPatch unnamedPatch from mid} {q : NamedPatch unnamedPatch
mid to}
              {q' : NamedPatch unnamedPatch from mid'} {p' : NamedPatch unnamed-
Patch mid' to}
              {q" : NamedPatch unnamedPatch from mid'} {p" : NamedPatch unnamed-
Patch mid' to},
      «p, q» <~> «q', p'»
   → «p, q» <~> «q", p"»
   → (p' = p") ∧ (q' = q").
Proof with auto.
intros u from mid mid' to p q q' p' q" p" commute' commute".
dependent destruction commute'.
dependent destruction commute".
assert (up' = up'0 ∧ uq' = uq'0).
apply (UnnamedPatchCommuteUnique u up uq)...
assert (p'OK = p'OK0).
apply proof_irrelevance.
assert (q'OK = q'OK0).
apply proof_irrelevance.
inversion H1.
subst...
Qed.
```

Explanation
This is Axiom 3.2 restated for named patches.

Proof
If $n(p) = n(q)$ or $n(p) = n(q)^{-1}$ then $j = k =$ fail.

Otherwise the result follows from Axiom 3.2 applied to the commute of the unnamed patches. ∎

Definition 7.6 (named-patches-commutable)
We extend $\underset{?}{\leftrightarrow}$ to relate two named patches if they are commutable, i.e.

$$p \underset{?}{\leftrightarrow} q \Leftrightarrow \exists p', q' \cdot \langle p, q \rangle \leftrightarrow \langle q', p' \rangle$$

```
Definition namedPatchCommutable {unnamedPatch : UnnamedPatch}
                                {from mid1 to : NameSet}
                                (p : NamedPatch unnamedPatch from mid1)
                                (q : NamedPatch unnamedPatch mid1 to) : Prop
 := ∃ mid2 : NameSet,
    ∃ q' : NamedPatch unnamedPatch from mid2,
    ∃ p' : NamedPatch unnamedPatch mid2 to,
    «p, q» <~> «q', p'».
Notation "p <~?~> q" := (namedPatchCommutable p q)
    (at level 60, no associativity).

Lemma namedPatchCommutable_dec :
      ∀ {unnamedPatch : UnnamedPatch}
            {from mid to : NameSet}
            (p : NamedPatch unnamedPatch from mid)
            (q : NamedPatch unnamedPatch mid to),
      {p <~?~> q} + {~(p <~?~> q)}.
Proof with auto.
intros.
dependent destruction p.
dependent destruction q.
rename np_name0 into snp.
rename np_name1 into snq.
rename np_unnamedPatch0 into up.
rename np_unnamedPatch1 into uq.
destruct (SignedNameOrderedTypeWithLeibniz.eq_dec snp snq).
    (*  This is the case where the two patches have the same name *)
    right.
    intro.
    dependent destruction H.
    dependent destruction H.
    dependent destruction H.
    dependent destruction H.
    apply SignedName_eq_leibniz in e.
    congruence.
destruct (SignedNameOrderedTypeWithLeibniz.eq_dec snp (signedNameInverse snq)).
    (*  This is the case where the two patches have inverse names *)
    right.
    intro.
    dependent destruction H.
    dependent destruction H.
    dependent destruction H.
    dependent destruction H.
    apply SignedName_eq_leibniz in e.
    congruence.
(*  In the remaining cases, whether the named patches commute is
   determined by whether the unnamed patches commute *)
destruct (unnamedPatchCommutable_dec unnamedPatch up uq).
    (*  This is the case where the unnamed patches commute *)
    left.
    unfold namedPatchCommutable.
    dependent destruction u.
    dependent destruction H.
```

```
rename x into up', x0 into uq'.
destruct snq.
rename s into sq, n1 into nq.
(*  XXX Can we get coq to leave this for us?: *)
set (snq := MkSignedName sq nq).
destruct snp.
rename s into sp, n1 into np.
(*  XXX Can we get coq to leave this for us?: *)
set (snp := MkSignedName sp np).
destruct sq.
    (*  This is the case where q is a positive patch *)
    remember (NameSetAdd nq from) as oq.
    ∃ oq.
    assert (q'OK : patchNamesOK from oq snq).
        destruct sp.
            simpl in ×.
            split.
                nameSetDec.
            nameSetDec.
        simpl in ×.
        subst. (*  XXX Shouldn't be needed *)
        split.
            admit.
            (*  nameSetDec. *)
        nameSetDec.
    ∃ (mkNamedPatch uq' snq q'OK).
    assert (p'OK : patchNamesOK oq to snp).
        destruct sp.
            simpl in ×.
            subst. (*  XXX Shouldn't be needed *)
            split.
                clear - np_nameOK0 n.
                admit. (*  nameSetDec. *)
            clear - np_nameOK0 np_nameOK1.
            nameSetDec.
        simpl in ×.
        subst. (*  XXX Shouldn't be needed *)
        split.
            clear - np_nameOK0 np_nameOK1 n0.
            admit. (*  nameSetDec. *)
        clear - np_nameOK0 np_nameOK1 n0.
        nameSetDec.
    ∃ (mkNamedPatch up' snp p'OK).
    apply MkNamedPatchCommute...
        admit.
    admit.
(*  This is the case where q is a negative patch *)
remember (NameSetRemove nq from) as oq.
∃ oq.
assert (q'OK : patchNamesOK from oq snq).
    destruct sp.
        simpl in ×.
```

```
            subst. (*  XXX Shouldn't be needed *)
            split.
                nameSetDec.
            admit. (*  nameSetDec. *)
        simpl in ×.
        subst. (*  XXX Shouldn't be needed *)
        split.
            nameSetDec.
        admit. (*  nameSetDec. *)
    ∃ (mkNamedPatch uq' snq q'OK).
    assert (p'OK : patchNamesOK oq to snp).
        destruct sp.
            simpl in ×.
            subst. (*  XXX Shouldn't be needed *)
            split.
                nameSetDec.
            destruct np_nameOK0 as [? HX1].
            destruct np_nameOK1 as [? HX2].
            destruct q'OK as [? HX3].
            assert (HX4 : NameSetIn nq from).
                clear - HX3.
                nameSetDec.
            clear - HX1 HX2 HX4 n0 H1.
            admit. (*  nameSetDec. *)
        simpl in ×.
        subst. (*  XXX Shouldn't be needed *)
        split.
            nameSetDec.
        destruct np_nameOK0 as [? HX1].
        destruct np_nameOK1 as [? HX2].
        clear - HX1 HX2 H0 H1 n.
        admit. (*  nameSetDec. *)
    ∃ (mkNamedPatch up' snp p'OK).
    apply MkNamedPatchCommute...
        admit.
    admit.
(*  This is the case where the unnamed patches do not commute *)
right.
intro.
dependent destruction H.
dependent destruction H.
dependent destruction H.
dependent destruction H.
assert (up <~?~>u uq).
    unfold unnamedPatchCommutable.
    ∃ up'.
    ∃ uq'...
congruence.
Qed.
```

Lemma 7.2 (named-patch-commute-self-inverse)
$\forall p \in \mathbf{P}, q \in \mathbf{P}, p' \in \mathbf{P}, q' \in \mathbf{P} \cdot$
$(\langle p, q\rangle \leftrightarrow \langle q', p'\rangle) \Leftrightarrow (\langle q', p'\rangle \leftrightarrow \langle p, q\rangle)$

```
Lemma NamedPatchCommuteSelfInverse :
      ∀ {unnamedPatch : UnnamedPatch}
            {from mid1 mid2 to : NameSet}
            {p : NamedPatch unnamedPatch from mid1}
            {q : NamedPatch unnamedPatch mid1 to}
            {q' : NamedPatch unnamedPatch from mid2}
            {p' : NamedPatch unnamedPatch mid2 to}
            (namedCommute : «p, q» <~> «q', p'»),
      («q', p'» <~> «p, q»).
Proof with auto.
intros.
destruct namedCommute.
apply UnnamedPatchCommuteSelfInverse in H.
apply MkNamedPatchCommute...
intro.
elim not_inverse.
apply namesInverseInjective.
rewrite nameInverseInverse...
Qed.
```

Explanation
This is Axiom 3.3 restated for named patches.

Proof
If $n(p) = n(q)$ or $n(p) = n(q)^{-1}$ then both commutes fail, so the result holds.

Otherwise the result follows from Axiom 3.3 applied to the commute of the unnamed patches. ∎

Lemma 7.3 (named-patch-commute-square)
$\forall p \in \mathbf{P}, q \in \mathbf{P}, \forall p' \in \mathbf{P}, q' \in \mathbf{P} \cdot$
$(\langle p, q\rangle \leftrightarrow \langle q', p'\rangle) \Leftrightarrow \left(\left\langle q'^{-1}, p\right\rangle \leftrightarrow \left\langle p', q^{-1}\right\rangle\right)$

Explanation
This is Axiom 3.4 restated for named patches.

Proof
If $n(p) = n(q)$ or $n(p) = n(q)^{-1}$ then both commutes fail, so the result holds.

Otherwise the result follows from Axiom 3.4 applied to the commute of the unnamed patches. ∎

```
(*  XXX Move this somewhere more general *)
Lemma monotonicNeq : ∀ {t u : Set}
                       (f : t → u)
                       (x y : t),
                     (f x ≠ f y)
                   → (x ≠ y).
Proof.
intros.
intro.
elim H.
```

```
  subst.
  auto.
  Qed.

  Lemma NamedPatchCommuteSquare :
        ∀ {unnamedPatch : UnnamedPatch}
                {o op oq opq : NameSet}
                {p : NamedPatch unnamedPatch o op}
                {q : NamedPatch unnamedPatch op opq}
                {q' : NamedPatch unnamedPatch o oq}
                {p' : NamedPatch unnamedPatch oq opq},
        «p, q» <~> «q', p'» →
        « q'^, p» <~> «p', q^».
  Proof with auto.
  intros.
  destruct H.
  apply UnnamedPatchCommuteSquare in H.
  constructor.
          simpl...
      simpl.
      apply (monotonicNeq signedNameInverse).
      rewrite nameInverseInverse.
      rewrite nameInverseInverse...
  simpl...
  Qed.
```

```
(*  XXX NamedPatchCommuteConsistent12 copy/pasted from
       patch_sequences without the accompanying text *)
Lemma NamedPatchCommuteConsistent1 :
      ∀ {unnamedPatch : UnnamedPatch}
              {o op opq opqr opr oq oqr : NameSet}
              {p1 : NamedPatch unnamedPatch o op}
              {q1 : NamedPatch unnamedPatch op opq}
              {r1 : NamedPatch unnamedPatch opq opqr}
              {q2 : NamedPatch unnamedPatch opr opqr}
              {r2 : NamedPatch unnamedPatch op opr}
              {q3 : NamedPatch unnamedPatch o oq}
              {r3 : NamedPatch unnamedPatch oq oqr}
              {p3 : NamedPatch unnamedPatch oqr opqr}
              {p5 : NamedPatch unnamedPatch oq opq},
      «q1, r1» <~> «r2, q2»
  → «p1, q1» <~> «q3, p5»
  → «p5, r1» <~> «r3, p3»
  → (∃ or : NameSet,
       (∃ r4 : NamedPatch unnamedPatch o or,
        (∃ q4 : NamedPatch unnamedPatch or oqr,
         (∃ p6 : NamedPatch unnamedPatch or opr,
          «q3, r3» <~> «r4, q4» ∧
          «p1, r2» <~> «r4, p6» ∧
          «p6, q2» <~> «q4, p3»)))).
Proof with auto.
```

```
intros unnamedPatch o op opq opqr opr oq oqr p1 q1 r1 q2 r2 q3 r3 p3 p5
       q1_r1_commutes_r2_q2
       p1_q1_commutes_q3_p5
       p5_r1_commutes_r2_p3.
dependent destruction q1_r1_commutes_r2_q2.
dependent destruction p1_q1_commutes_q3_p5.
dependent destruction p5_r1_commutes_r2_p3.
rename nq into nr.
rename np into nq.
rename np0 into np.
destruct np as [sp np].
destruct nq as [sq nq].
destruct nr as [sr nr].
(*  XXX Can we get coq to leave these for us?: *)
set (snr := MkSignedName sr nr).
set (snq4 := MkSignedName sq nq).
set (snp6 := MkSignedName sp np).
destruct (UnnamedPatchCommuteConsistent1 _ H H0 H1)
  as [r4 [q4 [p6 [H3 [H4 H5]]]]].
destruct sr.
    (*  This is the case where r is a positive patch *)
    remember (NameSetAdd nr o) as or.
    ∃ or.
    assert (snrPatchNamesOK : patchNamesOK o or snr).
        subst.
        simpl in ×.
        split.
            clear - q'OK pOK0 not_equal1 not_inverse1.
            destruct sp.
                nameSetDec.
            admit. (*  nameSetDec. *)
        clear.
        nameSetDec.
    ∃ (mkNamedPatch r4 snr snrPatchNamesOK).
    assert (q4PatchNamesOK : patchNamesOK or oqr snq4).
        subst.
        simpl in ×.
        destruct sq.
            split.
                clear - not_equal q'OK0.
                admit. (*  nameSetDec. *)
            clear - q'OK1 q'OK0.
            nameSetDec.
        split.
            clear - not_inverse q'OK1 q'OK0.
            admit. (*  nameSetDec. *)
        clear - not_inverse q'OK1 q'OK0.
        nameSetDec.
    ∃ (mkNamedPatch q4 snq4 q4PatchNamesOK).
    assert (p6PatchNamesOK : patchNamesOK or opr snp6).
        subst.
        simpl in ×.
```

```
        destruct sp.
            split.
                clear - pOK0 not_equal1.
                admit. (*  nameSetDec. *)
            clear - q'OK pOK0.
            nameSetDec.
        split.
            clear - pOK0 q'OK not_inverse1.
            admit. (*  nameSetDec. *)
        clear - pOK0 q'OK not_inverse1.
        nameSetDec.
    ∃ (mkNamedPatch p6 snp6 p6PatchNamesOK).
    split.
        apply MkNamedPatchCommute...
    split.
        apply MkNamedPatchCommute...
    apply MkNamedPatchCommute...
(*  This is the case where r is a negative patch *)
remember (NameSetRemove nr o) as or.
∃ or.
assert (snrPatchNamesOK : patchNamesOK o or snr).
    subst.
    simpl in ×.
    split.
        clear.
        nameSetDec.
    assert (HX : NameSetIn nr o).
        clear - q'OK pOK0 not_equal1 not_inverse1.
        destruct sp.
            admit. (*  nameSetDec. *)
        nameSetDec.
    clear - HX.
    admit. (*  nameSetDec. *)
∃ (mkNamedPatch r4 snr snrPatchNamesOK).
assert (q4PatchNamesOK : patchNamesOK or oqr snq4).
    subst.
    simpl in ×.
    destruct sq.
        split.
            clear - q'OK0.
            nameSetDec.
        clear - not_inverse q'OK1 q'OK0.
        admit. (*  nameSetDec. *)
    split.
        clear - q'OK1 q'OK0.
        nameSetDec.
    clear - q'OK1 q'OK0.
    admit. (*  nameSetDec. *)
∃ (mkNamedPatch q4 snq4 q4PatchNamesOK).
assert (p6PatchNamesOK : patchNamesOK or opr snp6).
    subst.
    simpl in ×.
```

```
    destruct sp.
        split.
            clear - pOK0.
            nameSetDec.
        clear - q'OK pOK0 not_inverse1.
        admit. (*  nameSetDec. *)
    split.
        clear - pOK0 q'OK.
        nameSetDec.
    clear - pOK0 q'OK.
    admit. (*  nameSetDec. *)
∃ (mkNamedPatch p6 snp6 p6PatchNamesOK).
split.
    apply MkNamedPatchCommute...
split.
    apply MkNamedPatchCommute...
apply MkNamedPatchCommute...
Qed.

Lemma NamedPatchCommuteConsistent2 :
      ∀ {unnamedPatch : UnnamedPatch}
              {o op opq opqr or oq oqr : NameSet}
              {q3 : NamedPatch unnamedPatch o oq}
              {r3 : NamedPatch unnamedPatch oq oqr}
              {p3 : NamedPatch unnamedPatch oqr opqr}
              {q4 : NamedPatch unnamedPatch or oqr}
              {r4 : NamedPatch unnamedPatch o or}
              {p1 : NamedPatch unnamedPatch o op}
              {q1 : NamedPatch unnamedPatch op opq}
              {r1 : NamedPatch unnamedPatch opq opqr}
              {p5 : NamedPatch unnamedPatch oq opq},
      «q3 , r3» <~> «r4 , q4»
   → «r3 , p3» <~> «p5 , r1»
   → «q3 , p5» <~> «p1 , q1»
   → (∃ opr : NameSet,
       (∃ r2 : NamedPatch unnamedPatch op opr ,
        (∃ q2 : NamedPatch unnamedPatch opr opqr ,
         (∃ p6 : NamedPatch unnamedPatch or opr ,
          «q1 , r1» <~> «r2 , q2» ∧
          «q4 , p3» <~> «p6 , q2» ∧
          «r4 , p6» <~> «p1 , r2»)))).
Proof with auto.
intros.
dependent destruction H.
dependent destruction H0.
dependent destruction H1.
rename nq into nr.
rename np into nq.
rename nq0 into np.
destruct np as [sp np].
destruct nq as [sq nq].
destruct nr as [sr nr].
(*  XXX Can we get coq to leave this for us?: *)
```

```
set (snr2 := MkSignedName sr nr).
set (snq2 := MkSignedName sq nq).
set (snp6 := MkSignedName sp np).
destruct (UnnamedPatchCommuteConsistent2 _ H H0 H1)
  as [ur2 [uq2 [up6 [? [? ?]]]]].
destruct sr.
    (*  This is the case where r is a positive patch *)
    remember (NameSetAdd nr op) as opr.
    ∃ opr.
    assert (snrPatchNamesOK : patchNamesOK op opr snr2).
        subst.
        simpl in ×.
        split.
            clear - q'OK q'OK1 not_equal0 not_inverse0.
            destruct sp.
                admit. (*  nameSetDec. *)
            nameSetDec.
        clear.
        nameSetDec.
    ∃ (mkNamedPatch ur2 snr2 snrPatchNamesOK).
    assert (q2PatchNamesOK : patchNamesOK opr opqr snq2).
        subst.
        simpl in ×.
        destruct sq.
            split.
                clear - not_equal p'OK1.
                admit. (*  nameSetDec. *)
            clear - p'OK0 p'OK1.
            nameSetDec.
        split.
            clear - not_inverse p'OK1 p'OK0.
            admit. (*  nameSetDec. *)
        clear - not_inverse p'OK1 p'OK0.
        nameSetDec.
    ∃ (mkNamedPatch uq2 snq2 q2PatchNamesOK).
    assert (p6PatchNamesOK : patchNamesOK or opr snp6).
        subst.
        simpl in ×.
        destruct sp.
            split.
                clear - q'OK q'OK1 not_equal0.
                admit. (*  nameSetDec. *)
            clear - q'OK q'OK1.
            nameSetDec.
        split.
            clear - q'OK1 not_inverse0.
            admit. (*  nameSetDec. *)
        clear - q'OK q'OK1 not_inverse0.
        nameSetDec.
    ∃ (mkNamedPatch up6 snp6 p6PatchNamesOK).
    split.
        apply MkNamedPatchCommute...
```

```
    split.
        apply MkNamedPatchCommute...
    apply MkNamedPatchCommute...
(*  This is the case where r is a negative patch *)
remember (NameSetRemove nr op) as opr.
∃ opr.
assert (snrPatchNamesOK : patchNamesOK op opr snr2).
    subst.
    simpl in ×.
    split.
        clear.
        nameSetDec.
    assert (HX : NameSetIn nr op).
        clear - q'OK1 q'OK not_equal0 not_inverse0.
        destruct sp.
            nameSetDec.
        admit. (*  nameSetDec. *)
    clear - HX.
    admit. (*  nameSetDec. *)
∃ (mkNamedPatch ur2 snr2 snrPatchNamesOK).
assert (q2PatchNamesOK : patchNamesOK opr opqr snq2).
    subst.
    simpl in ×.
    destruct sq.
        split.
            clear - p'OK1.
            nameSetDec.
        clear - p'OK0 p'OK1 not_inverse.
        admit. (*  nameSetDec. *)
    split.
        clear - p'OK1 p'OK0.
        nameSetDec.
    clear - p'OK1 p'OK0.
    admit. (*  nameSetDec. *)
∃ (mkNamedPatch uq2 snq2 q2PatchNamesOK).
assert (p6PatchNamesOK : patchNamesOK or opr snp6).
    subst.
    simpl in ×.
    destruct sp.
        split.
            clear - q'OK q'OK1.
            nameSetDec.
        clear - q'OK q'OK1 not_inverse0.
        admit. (*  nameSetDec. *)
    split.
        clear - q'OK1.
        nameSetDec.
    assert (HX : NameSetIn nr op).
        clear - snrPatchNamesOK.
            nameSetDec.
    clear - q'OK q'OK1 HX.
    admit. (*  nameSetDec. *)
```

```
∃ (mkNamedPatch up6 snp6 p6PatchNamesOK).
split.
    apply MkNamedPatchCommute...
split.
    apply MkNamedPatchCommute...
apply MkNamedPatchCommute...
Qed.

(*  XXX unused: *)
Lemma NamedPatchContexts : ∀ {unnamedPatch : UnnamedPatch}
                                    {from to : NameSet}
                                    (p : NamedPatch unnamedPatch from to),
                                patchNamesOK from to (np_name p).
Proof with auto.
intros.
destruct p.
unfold np_name.
unfold patchNamesOK in ×.
destruct np_name0.
destruct s; split; nameSetDec.
Qed.

Lemma NamedPatchCommuteNames :
      ∀ {unnamedPatch : UnnamedPatch}
            {from mid1 mid2 to : NameSet}
            {p : NamedPatch unnamedPatch from mid1} {q : NamedPatch unnamedPatch
mid1 to}
            {q' : NamedPatch unnamedPatch from mid2} {p' : NamedPatch unnamedPatch
mid2 to},
      « p , q » <~> « q' , p' »
   → (np_name p = np_name p') ∧
      (np_name q = np_name q') ∧
      (np_name p ≠ np_name q).
Proof with auto.
intros.
destruct H.
unfold np_name...
Qed.

Lemma NamedPatchInvertInverse :
      ∀ {unnamedPatch : UnnamedPatch}
            {from to : NameSet}
            (p : NamedPatch unnamedPatch from to),
      (p^)^ = p.
Proof with auto.
intros.
destruct p.
unfold namedPatchInverse.
simpl.
rewrite unnamedPatchInvertInverse.
destruct np_name0.
destruct s.
    simpl.
    f_equal.
```

```
    apply proof_irrelevance.
simpl.
f_equal.
apply proof_irrelevance.
Qed.

Definition NamedPartPatchUniverse (unnamedPatch : UnnamedPatch)
          : PartPatchUniverse (NamedPatch unnamedPatch) (NamedPatch unnamedPatch)
    := mkPartPatchUniverse
            (NamedPatch unnamedPatch)
            (NamedPatch unnamedPatch)
            (@namedPatchCommute _)
            (@namedPatchCommutable_dec _)
            (@NamedPatchCommuteUnique1 _)
            (@NamedPatchCommuteUnique2 _).

Definition NamedPatchUniverseInv
               {unnamedPatch : UnnamedPatch}
               (namedPartPatchUniverse : PartPatchUniverse
                                           (NamedPatch unnamedPatch)
                                           (NamedPatch unnamedPatch))
             : PatchUniverseInv (NamedPartPatchUniverse unnamedPatch)
                                (NamedPartPatchUniverse unnamedPatch)
    := mkPatchUniverseInv
            (NamedPatch unnamedPatch)
            (NamedPatch unnamedPatch)
            (NamedPartPatchUniverse unnamedPatch)
            (NamedPartPatchUniverse unnamedPatch)
            (@NamedPatchCommuteSelfInverse unnamedPatch).

Definition NamedPatchUniverse {unnamedPatch : UnnamedPatch}
                              {namedPartPatchUniverse : PartPatchUniverse
(NamedPatch unnamedPatch) (NamedPatch unnamedPatch)}
                              (namedPatchUniverseInv : PatchUniverseInv
                                                         (NamedPartPatchUniverse
unnamedPatch)
                                                         (NamedPartPatchUniverse
unnamedPatch))
 : PatchUniverse (NamedPatchUniverseInv namedPartPatchUniverse)
    := mkPatchUniverse
            (NamedPatch unnamedPatch)
            (NamedPartPatchUniverse unnamedPatch)
            (NamedPatchUniverseInv namedPartPatchUniverse)
            (@np_name _)
            (@NamedPatchCommuteConsistent1 _)
            (@NamedPatchCommuteConsistent2 _)
            (@NamedPatchCommuteNames _).

Definition NamedInvertiblePatchlike {unnamedPatch : UnnamedPatch}
                                    {namedPartPatchUniverse : PartPatchUniverse
(NamedPatch unnamedPatch) (NamedPatch unnamedPatch)}
                                    {namedPatchUniverseInv : PatchUniverseInv
                                                               (NamedPartPatchUniverse
unnamedPatch)
```

```
                                                    (NamedPartPatchUniverse
unnamedPatch)}
                                    (namedPatchUniverse : PatchUniverse
(NamedPatchUniverseInv namedPartPatchUniverse))
 : InvertiblePatchlike (NamedPatchUniverse namedPatchUniverseInv)
    := mkInvertiblePatchlike
         (NamedPatch unnamedPatch)
         (NamedPartPatchUniverse unnamedPatch)
         (NamedPatchUniverseInv namedPartPatchUniverse)
         (NamedPatchUniverse namedPatchUniverseInv)
         (@namedPatchInverse _)
         (@NamedPatchInvertInverse _)
         (@NamedPatchCommuteSquare _).

End NAMED_PATCHES.
```

8 Patch Merge

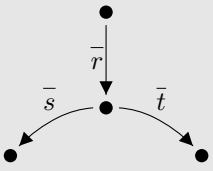
In order to give you some idea of where we are going, in this section we will describe how merging of non-conflicting patches works. This whole section is only informational.

We will give two descriptions on how merging works. The first, sequence-wise merge, is much simpler, but the second, patch-wise merge, is closer to what we have to do once conflicts are involved. Whichever merge algorithm is chosen, we first do merge preparation.

8.1 Merge preparation

Suppose we have two patch sequences (which could be entire repos) $\overline{p}$ and $\overline{q}$. They start from the same state (if they are repos, then the empty state), and then apply their respective patch sequences:

Then before we can merge these two repositories, we first need to commute their common patches to a prefix, i.e. we commute the patches within $\overline{p}$ and $\overline{q}$ to create:



where $\overline{p} \leftrightarrow^* \overline{rs}$, $\overline{q} \leftrightarrow^* \overline{rt}$, and $N\left(\overline{s}\right) \cap N\left(\overline{t}\right) = \emptyset$.

We will prove that it is always possible to commute $\overline{p}$ and $\overline{q}$ into such a state.

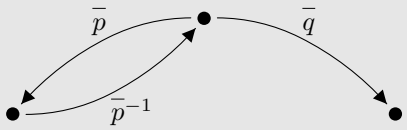
Then the actual merging is done on $\overline{s}$ and $\overline{t}$.

8.2 Sequence-wise merge

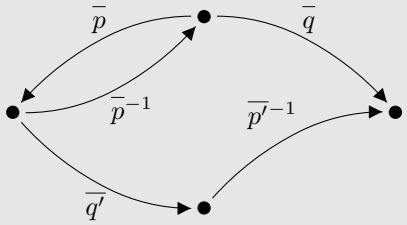
Suppose we have two sequences $\overline{p}$ and $\overline{q}$, where $N\left(\overline{p}\right) \cap N\left(\overline{q}\right) = \emptyset$:

Our goal is to find $\overline{p'}$ and $\overline{q'}$ such that $\overline{pq'} \leftrightarrow^* \overline{qp'}$, and is "morally equivalent" to applying both $\overline{p}$ and $\overline{q}$.

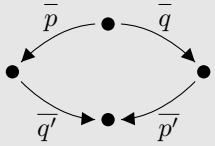
In order to achieve this, we use the fact that all patches are invertible, along with the notion of patch commute that we already have. To start with, let's add the sequence $\overline{p^{-1}}$ to our diagram:



Now $\overline{p}^{-1}$ and $\overline{q}$ are in sequence, so we can commute them:



Invert $\overline{p'}^{-1}$ again and throw away the inverted sequences, and we have the merge result:



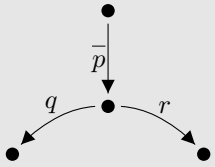
But does this satisfy the "moral equivalence" requirement?

Consider starting with the sequence $\overline{p}\overline{p}^{-1}\overline{q}$, which has the effect of just $\overline{q}$. Then we commuted $\overline{p}^{-1}$ and $\overline{q}$ giving us $\overline{pq'}\overline{p'}^{-1}$, which must still have the effect of just $\overline{q}$. The "moral equivalence" guaranteed by commute tells us that $\overline{p'}^{-1}$ must be "morally equivalent" to $\overline{p}^{-1}$, and thus $\overline{p'}^{-1}$ removes the "moral effect" of $\overline{p}$. Therefore, if we take our sequence $\overline{pq'}\overline{p'}^{-1}$ and throw away $\overline{p'}^{-1}$, leaving just $\overline{pq'}$, we must have the "moral effects" of $\overline{p}$ and $\overline{q}$.

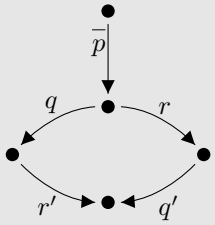
That paragraph was very hand-wavey, but that's because our notion of "moral equivalence" is very hand-wavey.

8.3 Patch-wise merge

To start with, let's see how to merge single patches. Suppose we have the two sequences $\overline{p}\overline{q}$ and $\overline{p}\overline{r}$, where $n\left(q\right) \neq n\left(r\right)$, i.e. they differ only in the last patch. We wish to merge these two sequences. We can make a graph containing the patches in both sequences thus:

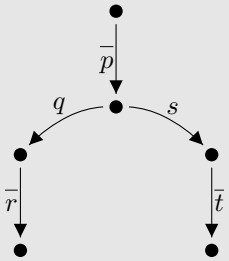


In order to merge the two sequences, we want a single sequence of patches incorporating the effects of both q and r. We do this in the same way that we handled sequences in sequence-wise merge, by completing the bottom of our diagram to form a commutation square:

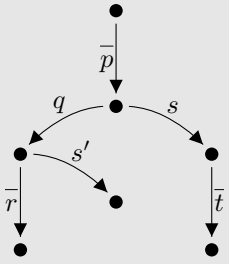


Provided the commute succeeds, we have that the result of the merge is qr' (or, equivalently, rq'). If the commute does not succeed then the patches cannot be cleanly merged, i.e. q and r conflict.

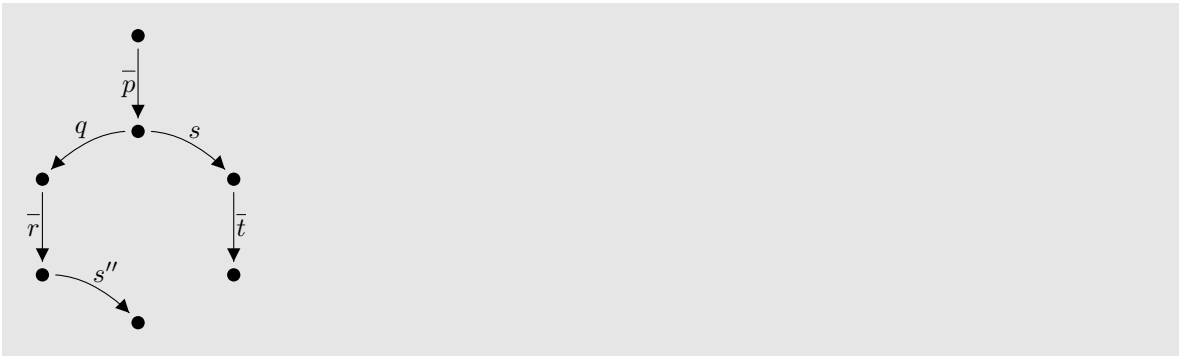
Now let's generalise to arbitrarily large merges. In one sequence we have $\overline{p}q\overline{r}$, and in the other we have $\overline{p}s\overline{t}$, where $N\left(q\overline{r}\right) \cap N\left(s\overline{t}\right) = \emptyset$:



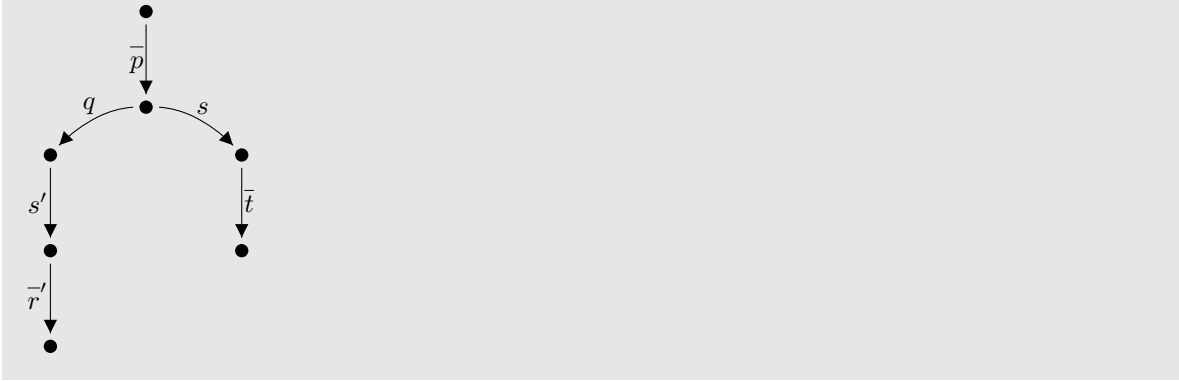
We start off by merging q and s:



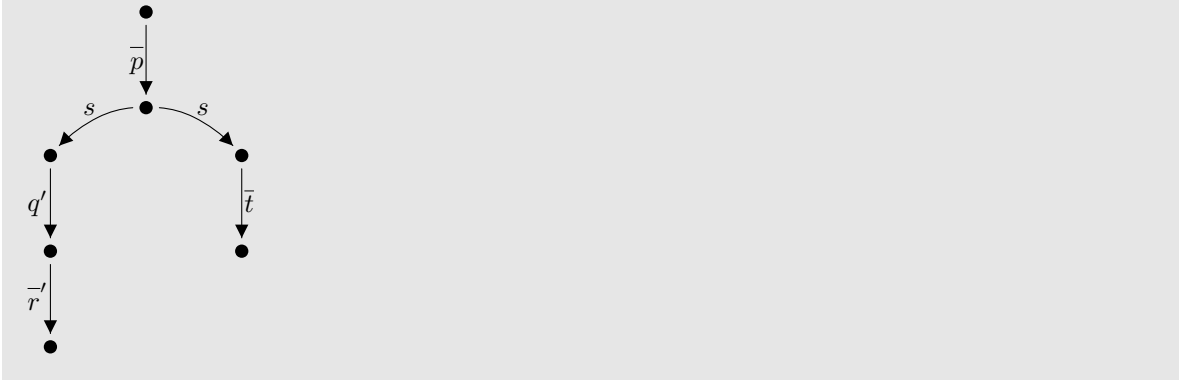
Note that qs' commutes to sq'. Now recursively merge s' with $\overline{r}$:



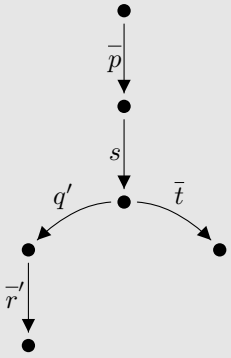
For the same reason that q and s' commute, s'' can be commuted past all the patches in $\overline{r}$:



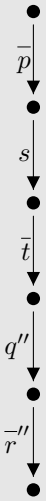
and of course, we can then commute s' and q:



Coalescing our two $\overline{p}s$ nodes gives us:



so we have merged s. Now we merge each patch in $\overline{t}$ in the same way, resulting in:



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printing $<\tilde{}>\mathrm{u} \leftrightsquigarrow_{\mathrm{u}}$ printing $<\tilde{}?\tilde{}>\mathrm{u} \overset{?}{\leftrightsquigarrow}_{\mathrm{u}}$ printing $\square\mathrm{u}\ \epsilon_{\mathrm{u}}$

printing $<\tilde{}> \leftrightsquigarrow$ printing $<\tilde{}?\tilde{}> \overset{?}{\leftrightsquigarrow}$ printing $\square\ \epsilon$ patch_universes

9 Patch Universes

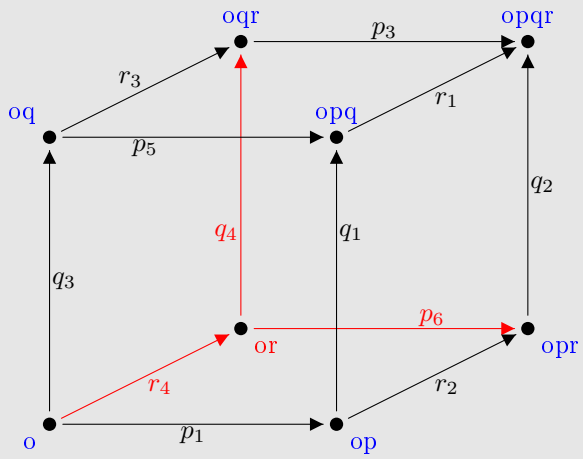
```
Module Export PATCH_UNIVERSES.
Ltac solveFirstIn hyp := refine (let v := _ in _ (hyp v));
                                 [ | clear hyp; intro hyp ].
Ltac solveExists :=
    match goal with
    | ⊢ ex (fun _ : ?y ⇒ _) ⇒ assert (seVar : y); [ | ∃ seVar ]
    end.
Require Import Equality.
Require Import names.
```

In this section we will introduce *patch universes*. Provided a patch type, and the operations on it, satisfies certain axioms, a set of sequences of patches forms a patch universe if it satisfies certain invariants.

In this section we will use notation normally used for named patches, but to refer to any patch type that can form a patch universe. Likewise, the term "patch" refers to the patch type of the particular patch universe being used. We use $\mathbb{U}$ to denote the entire universe.

```
Definition patchNamesOK (from to : NameSet)
                        (sn : SignedName)
                      : Prop
    := match sn with
       | MkSignedName Positive n ⇒ (¬ NameSetIn n from)
                                  ∧ (NameSetEqual to (NameSetAdd n from))
       | MkSignedName Negative n ⇒ (¬ NameSetIn n to)
                                  ∧ (NameSetEqual from (NameSetAdd n to))
       end.
Reserved Notation "« p , q » <~> « q' , p' »"
    (at level 60, no associativity).
Reserved Notation "p <~?~> q"
    (at level 60, no associativity).
Class PartPatchUniverse (pu_type1 pu_type2 : (NameSet → NameSet → Type))
                        : Type := mkPartPatchUniverse {
    commute : ∀ {from mid1 mid2 to : NameSet},
              pu_type1 from mid1 → pu_type2 mid1 to
           → pu_type2 from mid2 → pu_type1 mid2 to → Prop
        where "« p , q » <~> « q' , p' »" := (commute p q q' p');

    commutable : ∀ {from mid1 to : NameSet},
                 pu_type1 from mid1 → pu_type2 mid1 to → Prop
        := fun {from mid1 to : NameSet}
               (p : pu_type1 from mid1) (q : pu_type2 mid1 to) ⇒
           ∃ mid2 : NameSet,
           ∃ q' : pu_type2 from mid2,
           ∃ p' : pu_type1 mid2 to,
           «p, q» <~> «q', p'»
        where "p <~?~> q" := (commutable p q);

    commutable_dec : ∀ {from mid to : NameSet}
                       (p : pu_type1 from mid)
                       (q : pu_type2 mid to),
                     {p <~?~> q} + {~(p <~?~> q)};

    commuteUniqueTypes : ∀ {from mid mid' mid'' to : NameSet}
                           {p : pu_type1 from mid} {q : pu_type2 mid to}
                           {q' : pu_type2 from mid'} {p' : pu_type1 mid' to}
                           {q'' : pu_type2 from mid''} {p'' : pu_type1 mid'' to},
                         «p, q» <~> «q', p'»
                      → «p, q» <~> «q'', p''»
                      → (mid' = mid'');

    commuteUnique : ∀ {from mid mid' to : NameSet}
```

```
                                {p : pu_type1 from mid} {q : pu_type2 mid to}
                                {q' : pu_type2 from mid'} {p' : pu_type1 mid' to}
                                {q'' : pu_type2 from mid'} {p'' : pu_type1 mid' to},
                        «p, q» <~> «q', p'»
                    → «p, q» <~> «q'', p''»
                    → (p' = p'') ∧ (q' = q'')
}.
Notation "« p , q » <~> « q' , p' »" := (commute p q q' p').
Notation "p <~?~> q" := (commutable p q).

Class PatchUniverseInv {pu_type1 pu_type2 : (NameSet → NameSet → Type)}
                       (ppu1 : PartPatchUniverse pu_type1 pu_type2)
                       (ppu2 : PartPatchUniverse pu_type2 pu_type1)
                     : Type := mkPatchUniverseInv {
    commuteSelfInverse : ∀ {from mid1 mid2 to : NameSet}
                                {p : pu_type1 from mid1}
                                {q : pu_type2 mid1 to}
                                {q' : pu_type2 from mid2}
                                {p' : pu_type1 mid2 to},
        («p, q» <~> «q', p'»)
      → («q', p'» <~> «p, q»)
}.
```




```
Class PatchUniverse {pu_type : (NameSet → NameSet → Type)}
                    {ppu : PartPatchUniverse pu_type pu_type}
                    (pui : PatchUniverseInv ppu ppu)
                  : Type := mkPatchUniverse {
    pu_nameOf : ∀ {from to : NameSet}, pu_type from to → SignedName;

    commuteConsistent1 : ∀ {o op opq opqr opr oq oqr : NameSet}
                                {p1 : pu_type o op}
                                {q1 : pu_type op opq}
                                {r1 : pu_type opq opqr}
                                {q2 : pu_type opr opqr}
                                {r2 : pu_type op opr}
```

```
                                {q3 : pu_type o oq}
                                {r3 : pu_type oq oqr}
                                {p3 : pu_type oqr opqr}
                                {p5 : pu_type oq opq},
                           «q1 , r1» <~> «r2 , q2»
                        → «p1 , q1» <~> «q3 , p5»
                        → «p5 , r1» <~> «r3 , p3»
                        → ∃ or : NameSet,
                           ∃ r4 : pu_type o or,
                           ∃ q4 : pu_type or oqr,
                           ∃ p6 : pu_type or opr,
                           «q3 , r3» <~> «r4 , q4» ∧
                           «p1 , r2» <~> «r4 , p6» ∧
                           «p6 , q2» <~> «q4 , p3»;

    commuteConsistent2 : ∀ {o op opq opqr or oq oqr : NameSet}
                                {q3 : pu_type o oq}
                                {r3 : pu_type oq oqr}
                                {p3 : pu_type oqr opqr}
                                {q4 : pu_type or oqr}
                                {r4 : pu_type o or}
                                {p1 : pu_type o op}
                                {q1 : pu_type op opq}
                                {r1 : pu_type opq opqr}
                                {p5 : pu_type oq opq},
                           «q3 , r3» <~> «r4 , q4»
                        → «r3 , p3» <~> «p5 , r1»
                        → «q3 , p5» <~> «p1 , q1»
                        → ∃ opr : NameSet,
                           ∃ r2 : pu_type op opr,
                           ∃ q2 : pu_type opr opqr,
                           ∃ p6 : pu_type or opr,
                           «q1 , r1» <~> «r2 , q2» ∧
                           «q4 , p3» <~> «p6 , q2» ∧
                           «r4 , p6» <~> «p1 , r2»;

(*
This isn't true for conflictors, but happily turned out not to be used
in the proofs so far:
    pu_contexts : forall {from to : NameSet} (p : pu_type from to),
                  patchNamesOK from to (pu_nameOf p);
*)
```

We will write sequences of patches as $\overline{p}$, and we require that patches p have names $n(p)$. The set of names in a sequence of patches, or in a set of sequences of patches, is $N(p)$; the set of names used in the whole universe is therefore $N(\mathbb{U})$. The commute relations $\leftrightarrow$, $\underset{?}{\leftrightarrow}$ and $\leftrightarrow^*$ work on patches and sequences of patches in the normal way.

```
    commuteNames : ∀ {from mid1 mid2 to : NameSet}
                           {p : pu_type from mid1} {q : pu_type mid1 to}
                           {q' : pu_type from mid2} {p' : pu_type mid2 to},
                     « p , q » <~> « q' , p' »
                   → (pu_nameOf p = pu_nameOf p') ∧
                     (pu_nameOf q = pu_nameOf q') ∧
                     (pu_nameOf p ≠ pu_nameOf q)
}.
(*
Lemma commuteOKOfInverse : forall {o op op' opq o' oq oq' oqp : NameSet}
                                  (commuteOK : CommuteOK),
                           CommuteOK o' oq oq' oqp o op op' opq.
Proof.
intros.
constructor.
            remember commuteFromOK.
            clear - e.
            nameSetDec.
        remember commuteMid2OK.
        clear - e.
        nameSetDec.
    remember commuteMid1OK.
    clear - e.
    nameSetDec.
remember commuteToOK.
clear - e.
nameSetDec.
Qed.
*)
Class NilOK (from to : NameSet)
            (contains : SignedNameSet)
    := {
            nilFromEqTo : NameSetEqual from to;
            nilContainsNothing : SignedNameSetEqual
                                     contains
                                     SignedNameSetMod.empty
       }.
Class ConsOK {pu_type : NameSet → NameSet → Type}
             {ppu : PartPatchUniverse pu_type pu_type}
             {pui : PatchUniverseInv ppu ppu}
             {patchUniverse : PatchUniverse pui}
             {from' mid : NameSet}
             (from : NameSet)
             (contains tailContains : SignedNameSet)
             (head : pu_type from' mid)
    := {
            (*  consFromOK is a hack to make caseSequence/append work *)
            consFromOK : NameSetEqual from' from;
            consNotAlreadyThere : ¬ SignedNameSetIn (pu_nameOf head) tailContains;
            consContainsOK : SignedNameSetEqual
                                 contains
```

```
                                    (SignedNameSetAdd (pu_nameOf head) tailContains)
        }.

Inductive Sequence {pu_type : NameSet → NameSet → Type}
                   {ppu : PartPatchUniverse pu_type pu_type}
                   {pui : PatchUniverseInv ppu ppu}
                   (patchUniverse : PatchUniverse pui)
                   (contains : SignedNameSet)
                   (from to : NameSet)
                 : Type
    := Nil : ∀ (nilOK : NilOK from to contains),
              Sequence patchUniverse contains from to
     | Cons : ∀ {from' mid : NameSet}
                {qsContains : SignedNameSet}
                (p : pu_type from' mid)
                (qs : Sequence patchUniverse qsContains mid to)
                (consOK : ConsOK from contains qsContains p),
              Sequence patchUniverse contains from to.
Implicit Arguments Nil [pu_type ppu pui patchUniverse from to contains].
Implicit Arguments Cons [pu_type ppu pui patchUniverse from from' mid to contains qsCon-
tains].
Notation "s :> t" := (Cons s t _)
    (at level 60, right associativity).
Notation "[]" := (Nil _)
    (no associativity).

Program Definition castSequence {pu_type : NameSet → NameSet → Type}
                                {ppu : PartPatchUniverse pu_type pu_type}
                                {pui : PatchUniverseInv ppu ppu}
                                {patchUniverse : PatchUniverse pui}
                                {from from' to to' : NameSet}
                                {contains contains' : SignedNameSet}
                                (fromOK : NameSetEqual from from')
                                (toOK : NameSetEqual to to')
                                (containsOK : SignedNameSetEqual contains contains')
                                (ps : Sequence patchUniverse contains from to)
                              : Sequence patchUniverse contains' from' to'
    := match ps with
       | Nil _ ⇒ []
       | Cons _ _ _ p ps' _ ⇒ Cons p ps' _
       end.
Next Obligation.
constructor.
    remember nilFromEqTo as HX1.
    clear - HX1 fromOK toOK.
    nameSetDec.
remember nilContainsNothing as HX1.
clear - HX1 containsOK.
signedNameSetDec.
Qed.
Next Obligation.
apply NameSetEquality.
apply toOK.
```

```
Qed.
Next Obligation.
constructor.
        remember consFromOK as HX1.
        clear - HX1 fromOK.
        nameSetDec.
    apply consNotAlreadyThere.
rewrite ← containsOK.
apply consContainsOK.
Qed.

Class appendOK {pu_type : NameSet → NameSet → Type}
               {ppu : PartPatchUniverse pu_type pu_type}
               {pui : PatchUniverseInv ppu ppu}
               {patchUniverse : PatchUniverse pui}
               {from mid to : NameSet}
               {psContains qsContains : SignedNameSet}
               (ps : Sequence patchUniverse psContains from mid)
               (qs : Sequence patchUniverse qsContains mid to)
               (psQsContains : SignedNameSet)
    := {
        appendNoIntersection : SignedNameSetEqual
                                   (SignedNameSetMod.inter psContains qsContains)
                                   SignedNameSetMod.empty;
        appendContainsOK : SignedNameSetEqual
                               psQsContains
                               (SignedNameSetMod.union psContains qsContains)
       }.

Program Fixpoint append
               {pu_type : NameSet → NameSet → Type}
               {ppu : PartPatchUniverse pu_type pu_type}
               {pui : PatchUniverseInv ppu ppu}
               {patchUniverse : PatchUniverse pui}
               {from mid to : NameSet}
               {psContains qsContains psQsContains : SignedNameSet}
               (ps : Sequence patchUniverse psContains from mid)
               (qs : Sequence patchUniverse qsContains mid to)
               (thisAppendOK : appendOK ps qs psQsContains)
       : Sequence patchUniverse psQsContains from to
    := match ps with
       | Nil _ ⇒
           match qs with
           | Nil _ ⇒ []
           | Cons _ _ _ q qs' _ ⇒
               q :> qs'
           end
       | Cons _ ps'From ps'Contains p ps' _ ⇒
           p :> (append (from := ps'From) (to := to) (psQsContains := SignedNameSet-
Mod.union ps'Contains qsContains)
                        ps' qs _)
       end.
Next Obligation.
```

```
constructor.
    remember (nilFromEqTo (NilOK := wildcard')) as HX1.
    remember (nilFromEqTo (NilOK := wildcard'0)) as HX2.
    clear - HX1 HX2.
    nameSetDec.
remember (nilContainsNothing (NilOK := wildcard')) as HX1.
remember (nilContainsNothing (NilOK := wildcard'0)) as HX2.
remember appendContainsOK as HX3.
clear - HX1 HX2 HX3.
signedNameSetDec.
Qed.
Next Obligation.
constructor.
        remember consFromOK as HX1.
        remember nilFromEqTo as HX2.
        clear - HX1 HX2.
        nameSetDec.
    apply consNotAlreadyThere.
remember appendContainsOK as HX1.
remember consContainsOK as HX2.
remember nilContainsNothing as HX3.
clear - HX1 HX2 HX3.
signedNameSetDec.
Qed.
Next Obligation.
constructor.
    remember appendNoIntersection as HX1.
    remember consContainsOK as HX2.
    clear - HX1 HX2.
    signedNameSetDec.
reflexivity.
Qed.
Next Obligation.
constructor.
        apply consFromOK.
    remember consNotAlreadyThere as HX1.
    remember consContainsOK as HX2.
    remember appendNoIntersection as HX3.
    clear - HX1 HX2 HX3.
    signedNameSetDec.
remember consContainsOK as HX1.
remember appendContainsOK as HX2.
clear - HX1 HX2.
signedNameSetDec.
Qed.
Notation "ps :+> qs" := (append ps qs _)
    (at level 60, right associativity).
```

There is also a merge relation $\oplus$. Given two patches p and q, either there exists p' and q' such that $N(p) = N(p')$, $N(q) = N(q')$, $p \oplus q = \langle q', p' \rangle$ and $pq' \leftrightarrow^* qp'$; or $p \oplus q = \text{fail}$.

Axiom 9.1 (patch-universe-commute-preserves-commute)
$(\langle p, qr\rangle \leftrightarrow \langle q'r', p'\rangle) \Rightarrow \left(\left(q \underset{?}{\leftrightarrow} r\right) \Leftrightarrow \left(q' \underset{?}{\leftrightarrow} r'\right)\right)$

```
(*
    commute : pu_type -> pu_type -> pu_type -> pu_type -> Prop
        where "« p , q » <~> « q' , p' »" := (commute p q q' p');
*)
```

Explanation
This is Lemma ?? restated for patch universes.

Axiom 9.2 (patch-universe-commute-consistent)
$(\langle q, r\rangle \leftrightarrow \langle r', q'\rangle) \Rightarrow ((\langle p, qr\rangle \leftrightarrow \langle \_, p'\rangle) \Leftrightarrow (\langle p, r'q'\rangle \leftrightarrow \langle \_, p'\rangle))$

Explanation
This is Lemma ?? restated for named patches.

Those are the operations and axioms that your patch type must provide. We now explain what can be done in a patch universe, and what invariants must be satisfied by the operations while they are done.

The operations you can do in a patch universe are:

Record Given a sequence of patches $\overline{p}$ we can record a patch with a fresh name, giving us $\overline{pq}$.

Unrecord Given a sequence of patches $\overline{pq}$ we can unpull q leaving us $\overline{p}$.

Commute Given $\overline{p}$, we can make $\overline{q}$ where $\overline{p} \leftrightarrow^* \overline{q}$.

Merge Given $\overline{pq}$ and $\overline{pr}$, if $\overline{q} \oplus \overline{r} = \left\langle \overline{r'}, \overline{q'} \right\rangle$ then we can make $\overline{pqr'}$ and $\overline{prq'}$.

XXX and unrecord

XXX The remainder of this section is proofs of things

Lemma 9.1 (commute-in-sequence-consistent)
Suppose $\overline{p}qr\overline{s} \leftrightarrow^* \overline{t}q'r'\overline{u}$ where $n(q) = n(q')$ and $n(r) = n(r')$. Then $\left(q \underset{?}{\leftrightarrow} r\right) \Leftrightarrow \left(q' \underset{?}{\leftrightarrow} r'\right)$.

Explanation
Suppose we have adjacent two patches q and r in a sequence. No amount of commuting within this sequence can alter whether or not q and r commute.

Proof
Suppose, for the purpose of contradiction, that we have a counter example of minimal sequence length. Without loss of generality, assume that $q \underset{?}{\leftrightarrow} r$ holds but $q' \underset{?}{\leftrightarrow} r'$ does not.

If $\overline{p} = \epsilon$ and $\overline{s} = \epsilon$ then we trivially have a contradiction. We will assume that $\overline{p}$ is non-empty, but an analogous argument covers the case where only $\overline{s}$ is non-empty.

So $\overline{p} = v\overline{p'}$, for some patch v and sequence $\overline{p'}$. We will first show, by induction on the structure of $\leftrightarrow^*$, that at each step v can be commuted directly to the start of the sequence.

Trivially that is true for the initial sequence $v\overline{p'}qr\overline{s}$. Suppose that it is true for a sequence $\overline{a_1}a_2a_3\overline{a_4}$ where $\langle a_2, a_3\rangle \leftrightarrow \langle a_3', a_2'\rangle$; then we must show that it is true for $\overline{a_1}a_3'a_2'\overline{a_4}$. If v is in $\overline{a_1}$ then it is trivially true, using the same sequence of commutes as before. If v is a_2 then it is true, by first commuting with a_3' and then continuing as before. If v is a_3 then it is true, by skipping the first commute of the previous sequence. The must interesting case is when v is in $\overline{a_4}$. We know that v can be commuted past everything else in $\overline{a_4}$ until

we have $\overline{a_1}a_2a_3v'\overline{a'_4}$, then past a_2a_3 giving us $\overline{a_1}v''a''_2a''_3\overline{a'_4}$, and finally past $\overline{a_1}$, giving us $v'''\overline{a'_1}a''_2a''_3\overline{a'_4}$. But Axiom 9.2 tells us that $\langle a'_3a'_2, v'\rangle \leftrightarrow \langle v'', a'''_3a'''_2\rangle$, so it is also true for this case.

Now, consider the final sequence. If v is in $\overline{t}$ then trivially whether q' and r' commute is unchanged. If v is in $\overline{u}$ then Axiom 9.1 tells us that whether q' and r' commute is unchanged.

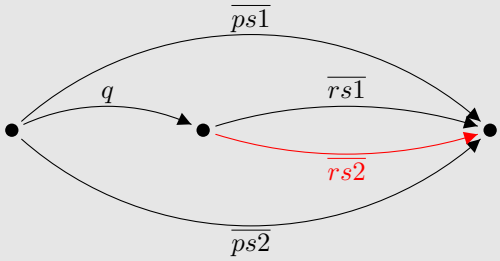
Therefore we can construct a shorter counter example, by commuting v to the left of the sequence at each stage, and removing it. ■

```
(*
XXX
Lemma NamePersistsInSequenceNameSet :
      forall {patchUniverse : PatchUniverse}
             {o op opq opqr : NameSet}
             (p : (pu_type patchUniverse) o op)
             (qs : Sequence (pu_type patchUniverse) op opq)
             (rs : Sequence (pu_type patchUniverse) opq opqr),
      NameSetIn (pu_nameOf patchUniverse p) opq
   -> NameSetIn (pu_nameOf patchUniverse p) opqr.
Proof with auto.
intros.
dependent induction rs...
specialize (IHrs patchUniverse).
specialize (IHrs p).
specialize (IHrs mid).
specialize (IHrs (qs :+> (p0 :> ))).
assert (p_in_mid : NameSetIn (pu_nameOf patchUniverse p) mid).
    destruct (pu_contexts patchUniverse p0).
    fsetdec.
specialize (IHrs p_in_mid).
specialize (IHrs opqr refl refl)...
Qed.

Lemma NamePersistsInSequence :
      forall {patchUniverse : PatchUniverse}
             {o op opq : NameSet}
             (p : (pu_type patchUniverse) o op)
             (qs : Sequence (pu_type patchUniverse) op opq),
      NameSetIn (pu_nameOf patchUniverse p) opq.
Proof with auto.
intros.
assert (p_in_op : NameSetIn (pu_nameOf patchUniverse p) op).
    destruct (pu_contexts patchUniverse p).
    fsetdec.
set (H1 := NamePersistsInSequenceNameSet p  qs p_in_op)...
Qed.

Lemma NameInSequenceUnique :
      forall {patchUniverse : PatchUniverse}
             {o op opq opqr : NameSet}
             (p : (pu_type patchUniverse) o op)
             (qs : Sequence (pu_type patchUniverse) op opq)
```

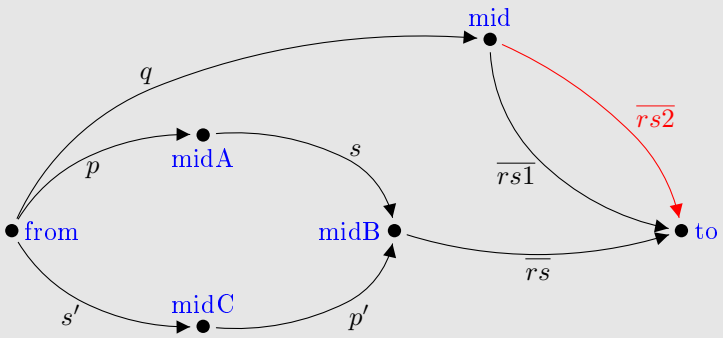
```
            (r : (pu_type patchUniverse) opq opqr),
      (pu_nameOf patchUniverse p <> pu_nameOf patchUniverse r).
Proof with auto.
intros.
set (p_in_opq := NamePersistsInSequence p qs).
clearbody p_in_opq.
destruct (pu_contexts patchUniverse r).
congruence.
Qed.
*)

Lemma commuteRelatesLemma1 {x y : SignedName}
                           {zs : SignedNameSet}
                         : (y ≠ x)
                        → ¬ SignedNameSetIn x zs
                        → ¬ SignedNameSetIn x (SignedNameSetAdd y zs).
Proof.
intros.
admit.
(*
XXX coq 2691 *)
signedNameSetDec.
Qed.

Lemma TransitiveTransitiveCommute :
    ∀ {pu_type : NameSet → NameSet → Type}
          {ppu : PartPatchUniverse pu_type pu_type}
          {pui : PatchUniverseInv ppu ppu}
          {patchUniverse : PatchUniverse pui}
          {from to : NameSet}
          {contains : SignedNameSet}
          {ps qs rs : Sequence patchUniverse contains from to}
          (ps_qs : «ps» <~~>* «qs»)
          (qs_rs : «qs» <~~>* «rs»),
    («ps» <~~>* «rs»).
Proof with auto.
intros.
induction ps_qs...
apply IHps_qs in qs_rs.
apply (Swap H qs_rs).
Qed.

Lemma SymmetricCommute :
    ∀ {pu_type : NameSet → NameSet → Type}
          {ppu : PartPatchUniverse pu_type pu_type}
          {pui : PatchUniverseInv ppu ppu}
          {patchUniverse : PatchUniverse pui}
          {from to : NameSet}
          {contains : SignedNameSet}
          {ps qs : Sequence patchUniverse contains from to}
          (ps_qs : «ps» <~~> «qs»),
    («qs» <~~> «ps»).
Proof with auto.
intros.
```

```
induction ps_qs.
    apply commuteSelfInverse in H.
    apply (SwapNow _ _ _ _ H).
apply SwapLater...
Qed.

Lemma SymmetricTransitiveCommute :
    ∀ {pu_type : NameSet → NameSet → Type}
            {ppu : PartPatchUniverse pu_type pu_type}
            {pui : PatchUniverseInv ppu ppu}
            {patchUniverse : PatchUniverse pui}
            {from to : NameSet}
            {contains : SignedNameSet}
            {ps qs : Sequence patchUniverse contains from to}
            (ps_qs : «ps» <~~>* «qs»),
    («qs» <~~>* «ps»).
Proof with auto.
intros.
induction ps_qs.
    apply Same.
apply SymmetricCommute in H.
apply (TransitiveTransitiveCommute IHps_qs (Swap H (Same _))).
Qed.

Lemma EnlargeTransitiveCommuteRelation :
    ∀ {pu_type : NameSet → NameSet → Type}
            {ppu : PartPatchUniverse pu_type pu_type}
            {pui : PatchUniverseInv ppu ppu}
            {patchUniverse : PatchUniverse pui}
            {from mid to : NameSet}
            {contains contains' : SignedNameSet}
            {p : pu_type from mid}
            {qs rs : Sequence patchUniverse contains' mid to}
            (pTailConsOK : ConsOK from contains contains' p)
            (qs_rs : «qs» <~~>* «rs»),
    (« (p :> qs) » <~~>* « (p :> rs) »).
Proof with auto.
intros.
induction qs_rs.
    apply Same.
apply (Swap (SwapLater p _ H) IHqs_rs).
Qed.

Lemma commuteOutLeftConsistentStep :
    ∀ {pu_type : NameSet → NameSet → Type}
            {ppu : PartPatchUniverse pu_type pu_type}
            {pui : PatchUniverseInv ppu ppu}
            {patchUniverse : PatchUniverse pui}
            {from mid to : NameSet}
            {contains contains' : SignedNameSet}
            {ps1 ps2 : Sequence patchUniverse contains from to}
            {q : pu_type from mid}
            {rs1 : Sequence patchUniverse contains' mid to}
            (commute_relates_ps1_ps2 : «ps1» <~~> «ps2»)
```

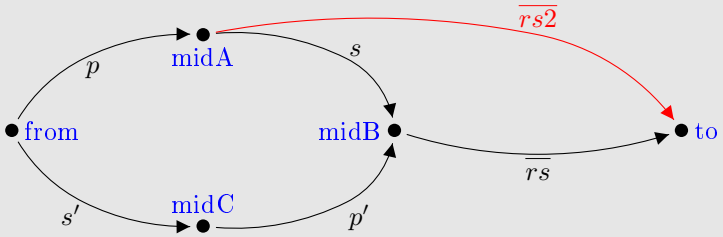
```
        (q_rs1_commutes_left_out_of_ps1 : «q, rs1» <~ «ps1»),
    (∃ rs2 : Sequence patchUniverse contains' mid to,
     («rs1» <~~>* «rs2») ∧
     («q, rs2» <~ «ps2»)).
Proof with auto.
intros.
```



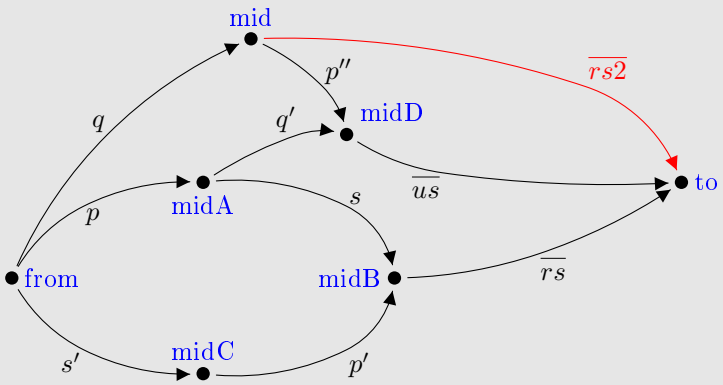

```
dependent induction commute_relates_ps1_ps2
  (*  I think needing the generalizing clause is a coq bug.
     Without it, mid doesn't get generalised so we can't do
     inductive case. *)
  generalizing mid q contains' rs1
               q_rs1_commutes_left_out_of_ps1.
    (*  This is the case where we have a swap at the head *)
    rename q0 into s, q' into s'.
    rename op into midA, opq into midB, oq into midC.
    rename H into p_s_commute_s'_p'.
```



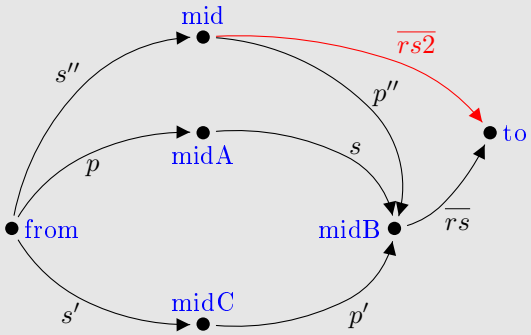

```
    dependent destruction q_rs1_commutes_left_out_of_ps1.
        (*  This is the case where the commute out is done.
           But the swap has messed it up, so we will have to
           undo it to build the commute out. *)
```



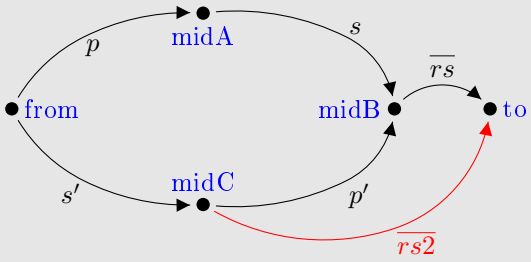

```
    ∃ (s :> rs).
    split.
        apply Same.
    apply commuteSelfInverse in p_s_commute_s'_p'.
    refine (commuteOutLeftSwap _ _ _ p_s_commute_s'_p')...
    apply commuteOutLeftDone.
(*  This is the case where the commuteout actually does some work *)
rename mid_q into midD.
rename q into p", p0 into q', p'0 into q, qs into us.
rename H into p_q'_commute_q_p".
```




```
(*  There are two possibilities:
   * The commuteout does exactly one commute, and the swap has
     just done it for us
   * The commuteout does more than one swap, and so we need to
     use the lemma to rewrite the last 2 swaps *)
dependent destruction q_rs1_commutes_left_out_of_ps1.
    (*  This is the case where the commuteout does
        exactly one commute, so we now have nothing to do *)
    rename q into s".
    rename p_q'_commute_q_p" into p_s_commute_s"_p".
```

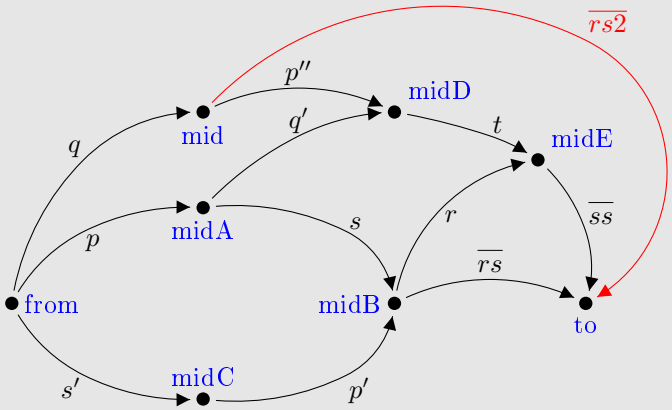



```
set (H := commuteUniqueTypes p_s_commute_s'_p' p_s_commute_s"_p").
clearbody H.
subst.
subst.
set (H := commuteUnique p_s_commute_s'_p' p_s_commute_s"_p").
clearbody H.
destruct H.
subst.
rename p" into p', s" into s'.
rename mid into midC.
```




```
∃ (p' :> rs).
split.
    apply Same.
assert (Heq : SignedNameSetEqual contains' pRsContains).
    remember (consContainsOK (contains := contains')) as H1.
    remember (consContainsOK (contains := pRsContains)) as H2.
    clear - H1 H2.
    signedNameSetDec.
apply SignedNameSetEquality in Heq.
subst.
assert (Heq : pRsConsOK = q_qsConsOK).
    apply proof_irrelevance.
subst.
apply commuteOutLeftDone.
(*  This is the case where the commuteout does
```

```
      at least commute2, so we have to jump over the swap *)
    rename mid_q into midE.
    rename p'0 into q'.
    rename p0 into r.
    rename q0 into t.
    rename qs into ss.
    rename H into s_r_commute_q'_t.
    rename qsContains into t_ssContains, qsContains0 into ssContains.
```




```
    assert (H := commuteConsistent2
                   p_s_commute_s'_p'
                   s_r_commute_q'_t
                   p_q'_commute_q_p").
    destruct H as [? [t' [p"' [r'
                  [p"_t_commute_t'_p"'
                   [p'_r_commute_r'_p"'
                    s'_r'_commute_q_t']]]]]].
    destruct (commuteNames p"_t_commute_t'_p"')
          as [name_p"_name_p"'
              [name_t_name_t'
               name_p_not_name_t]].
    assert (p"'_ssConsOK : ConsOK x (SignedNameSetAdd (pu_nameOf p"') ssContains)
ssContains p"').
        constructor.
            reflexivity.
          remember (consNotAlreadyThere (ConsOK := q_qsConsOK)) as HX1.
          rewrite ← name_p"_name_p"'.
          remember (consContainsOK (ConsOK := q_qsConsOK0)) as HX2.
          clear - HX1 HX2.
          signedNameSetDec.
        reflexivity.
    assert (t'_p"'_ssConsOK : ConsOK mid contains' (SignedNameSetAdd (pu_nameOf p"')
ssContains) t').
        constructor.
            reflexivity.
```

```
                remember (consNotAlreadyThere (ConsOK := q_qsConsOK0)) as HX1.
                rewrite ← name_t_name_t'.
                rewrite ← name_p"_name_p"'.
                clear - HX1 name_p_not_name_t.
                remember (pu_nameOf t) as HX2. (*  Workaround for coq 2464 *)
                admit.
                (*
                 XXX coq 2464 *)
                admit.
                (*
                 XXX coq 2464 *)
      remember (pu_nameOf r) as HX5. (*  Workaround for coq 2467
      signedNameSetDec.
      *)
assert (p_not_r' : pu_nameOf p ≠ pu_nameOf r').
      rewrite ← rs_same...
assert (p_not_s : pu_nameOf p ≠ pu_nameOf s).
      remember (consNotAlreadyThere (ConsOK := p_qs_r_s_tsConsOK)) as HX1.
      remember (appendContainsOK (appendOK := qs_r_s_tsAppendOK)) as HX2.
      remember (consContainsOK (ConsOK := r_s_tsConsOK)) as HX3.
      remember (consContainsOK (ConsOK := s_tsConsOK)) as HX4.
      clear - HX1 HX2 HX3 HX4.
      remember (pu_nameOf p) as HX5. (*  Workaround for coq 2464 *)
      remember (pu_nameOf s) as HX7. (*  Workaround for coq 2467
      signedNameSetDec.
      *)
assert (p_not_s' : pu_nameOf p ≠ pu_nameOf s').
      rewrite ← ss_same...
set (HX := CommuteOutLeftPreservesCommute
                    _ _ _ p_not_r' p_not_s' p_out_of_rhs).
destruct HX as [opp [oppq [oppqr [ps"Contains [ss"Contains
                  [ps" [q" [r" [ss"
                  [r"_ss"Contains [q"_r"_ss"Contains
                  [r"_ss"ConsOK [q"_r"_ss"ConsOK
                  [ps"_q"_r"_ss"AppendOK
                  [? [? [? ?]]]
]]] ]]]]
]]
]]]]].
∃ opp.
∃ oppq.
∃ oppqr.
∃ ps"Contains.
∃ ss"Contains.
∃ ps".
∃ q".
∃ r".
∃ ss".
∃ r"_ss"Contains.
∃ q"_r"_ss"Contains.
∃ r"_ss"ConsOK.
∃ q"_r"_ss"ConsOK.
```

```
∃ ps"_q"_r"_ss"AppendOK.
subst.
split...
split.
rewrite rs_same...
rewrite ss_same...
Qed.
Lemma EmptyCounterExampleExists :
    ∀ {pu_type : NameSet → NameSet → Type}
            {ppu : PartPatchUniverse pu_type pu_type}
            {pui : PatchUniverseInv ppu ppu}
            {patchUniverse : PatchUniverse pui}
            {o oq oqr oqrs ou our ours : NameSet}
            {qsContains usContains vsContains : SignedNameSet}
            {qs : Sequence patchUniverse qsContains o oq}
            {r : pu_type oq oqr}
            {s : pu_type oqr oqrs}
            {us : Sequence patchUniverse usContains o ou}
            {r' : pu_type ou our}
            {s' : pu_type our ours}
            {vs : Sequence patchUniverse vsContains ours oqrs}

            {NilContains s_NilContains r_s_NilContains qs_r_s_NilContains : SignedName-
Set}
            (nilOK : NilOK oqrs oqrs NilContains)
            (s_NilConsOK : ConsOK oqr s_NilContains NilContains s)
            (r_s_tsConsOK : ConsOK oq r_s_NilContains s_NilContains r)
            (qs_r_s_NilAppendOK : appendOK qs (r :> s :> []) qs_r_s_NilContains)

            {s'_vsContains r'_s'_vsContains : SignedNameSet}
            (s'_vsConsOK : ConsOK our s'_vsContains vsContains s')
            (r'_s'_vsConsOK : ConsOK ou r'_s'_vsContains s'_vsContains r')
            (us_r'_s'_vsAppendOK : appendOK us (r' :> s' :> vs) qs_r_s_NilContains)

            (*  t omitted for now *)
            (transitive_commute : «qs :+> r :> s :> []» <~~>* «us :+> r' :> s' :> vs»)
            (rs_same : pu_nameOf r = pu_nameOf r')
            (ss_same : pu_nameOf s = pu_nameOf s')
            (r_s_commutable : r <~?~> s)
            (r'_s'_not_commutable : ~(r' <~?~> s')),
    (
     ∃ lNilOK : NilOK oq oq SignedNameSetMod.empty,
     ∃ lNil_r_s_NilAppendOK : appendOK [] (r :> s :> []) r_s_NilContains,
     ∃ ou' : NameSet,
     ∃ our' : NameSet,
     ∃ ours' : NameSet,
     ∃ us'Contains : SignedNameSet,
     ∃ vs'Contains : SignedNameSet,
     ∃ us' : Sequence patchUniverse us'Contains oq ou',
     ∃ r": pu_type ou' our',
     ∃ s": pu_type our' ours',
     ∃ vs' : Sequence patchUniverse vs'Contains ours' oqrs,
```

```
      ∃ s"_vs'Contains : SignedNameSet,
      ∃ r"_s"_vs'Contains : SignedNameSet,
      ∃ s"_vs'ConsOK : ConsOK our' s"_vs'Contains vs'Contains s",
      ∃ r"_s"_vs'ConsOK : ConsOK ou' r"_s"_vs'Contains s"_vs'Contains r",
      ∃ us'_r"_s"_vs'AppendOK : appendOK us' (r" :> s" :> vs') r_s_NilContains,
          TransitiveCommuteRelates ([] :+> r :> s :> [])
                                   (us' :+> r" :> s" :> vs') ∧
          (pu_nameOf r = pu_nameOf r') ∧
          (pu_nameOf s = pu_nameOf s') ∧
          ~ (r" <~?~> s")).
Proof with trivial.
intros.
move qs at top.
revert oqr oqrs ou our ours usContains vsContains r s us r' s' vs
s_NilContains r_s_NilContains qs_r_s_NilContains nilOK s_NilConsOK r_s_tsConsOK
qs_r_s_NilAppendOK s'_vsContains r'_s'_vsContains s'_vsConsOK r'_s'_vsConsOK
us_r'_s'_vsAppendOK transitive_commute rs_same ss_same r_s_commutable
r'_s'_not_commutable.
dependent induction qs.
    (*   case *)
    simpl.
    intros.
    assert (Heq : from = to).
        apply NameSetEquality.
        apply nilFromEqTo.
    subst.
    assert (Heq : contains = SignedNameSetMod.empty).
        apply SignedNameSetEquality.
        apply nilContainsNothing.
    subst.
    assert (Heq : r_s_NilContains = qs_r_s_NilContains).
        apply SignedNameSetEquality.
        remember (appendContainsOK (appendOK := qs_r_s_NilAppendOK)) as HX1.
        clear - HX1.
        signedNameSetDec.
    subst.
    ∃ nilOK.
    ∃ qs_r_s_NilAppendOK.
    ∃ ou.
    ∃ our.
    ∃ ours.
    ∃ usContains.
    ∃ vsContains.
    ∃ us.
    ∃ r'.
    ∃ s'.
    ∃ vs.
    ∃ s'_vsContains.
    ∃ r'_s'_vsContains.
    ∃ s'_vsConsOK.
    ∃ r'_s'_vsConsOK.
    ∃ us_r'_s'_vsAppendOK.
```

```
    split.
        apply transitive_commute.
    repeat split...
(*  Cons case *)
simpl.
intros.
assert (from' = from).
    apply NameSetEquality.
    apply consFromOK.
subst.
set (HX := ShorterCounterExampleExists _ _ _ _ _ _ _ transitive_commute rs_same ss_same
r_s_commutable r'_s'_not_commutable).
destruct HX as [ou' [our' [ours'
                [us'Contains [vs'Contains
                [us' [r" [s" [vs'
                [s"_vs'Contains [r"_s"_vs'Contains
                [s"_vs'ConsOK [r"_s"_vs'ConsOK [us'_r"_s"_vs'AppendOK
                [transitive_commute_3 [r_r"_same [s_s"_same r"_s"_not_commutable
                ]]]]]]]]]]]]]]]].
specialize (IHqs NilContains).
specialize (IHqs oqr oqrs).
specialize (IHqs ou' our' ours').
specialize (IHqs us'Contains vs'Contains).
specialize (IHqs r s).
specialize (IHqs us' r" s" vs').
specialize (IHqs s_NilContains).
specialize (IHqs r_s_NilContains).
specialize (IHqs (SignedNameSetMod.union qsContains r_s_NilContains)).
specialize (IHqs nilOK).
specialize (IHqs s_NilConsOK).
specialize (IHqs r_s_tsConsOK).
specialize (IHqs (append_obligation_3 pu_type ppu pui patchUniverse from to oqrs contains
          r_s_NilContains qs_r_s_NilContains (Cons p qs consOK)
          (Cons r (Cons s (Nil nilOK) s_NilConsOK) r_s_tsConsOK)
          qs_r_s_NilAppendOK from mid qsContains p qs consOK eq_refl)).
specialize (IHqs s"_vs'Contains).
specialize (IHqs r"_s"_vs'Contains).
specialize (IHqs s"_vs'ConsOK).
specialize (IHqs r"_s"_vs'ConsOK).
specialize (IHqs us'_r"_s"_vs'AppendOK).
specialize (IHqs transitive_commute_3).
specialize (IHqs r_r"_same s_s"_same).
specialize (IHqs r_s_commutable r"_s"_not_commutable).
simpl in IHqs.
destruct IHqs as [lNilOK [lNil_r_s_NilAppendOK
                [ou" [our" [ours"
                [us"Contains [vs"Contains
                [us" [r"' [s"' [vs"
                [s"'_vs"Contains
                [r"'_s"'_vs"Contains
                [s"'_vs"ConsOK
                [r"'_s"'_vs"ConsOK
```

```
                        [us"_r"'_s"'_vs"AppendOK
                         [? [? [? ?]]]
]]]]]]]]]]]]]]].
∃ lNilOK.
∃ lNil_r_s_NilAppendOK.
∃ ou".
∃ our".
∃ ours".
∃ us"Contains.
∃ vs"Contains.
∃ us".
∃ r"'.
∃ s"'.
∃ vs".
∃ s"'_vs"Contains.
∃ r"'_s"'_vs"Contains.
∃ s"'_vs"ConsOK.
∃ r"'_s"'_vs"ConsOK.
∃ us"_r"'_s"'_vs"AppendOK.
repeat split...
Qed.

Lemma TwoSequenceTransitiveCommuteRelatesTwoSequence1 :
    ∀ {pu_type : NameSet → NameSet → Type}
            {ppu : PartPatchUniverse pu_type pu_type}
            {pui : PatchUniverseInv ppu ppu}
            {patchUniverse : PatchUniverse pui}
            {o or ors ou our ours : NameSet}
            {usContains vsContains : SignedNameSet}
            {r : pu_type o or}
            {s : pu_type or ors}
            {us : Sequence patchUniverse usContains o ou}
            {r' : pu_type ou our}
            {s' : pu_type our ours}
            {vs : Sequence patchUniverse vsContains ours ors}

            {RNilContains s_RNilContains r_s_RNilContains : SignedNameSet}
            {LNilContains : SignedNameSet}
            {s'_vsContains r'_s'_vsContains : SignedNameSet}
            {contains : SignedNameSet}

            {LNilOK : NilOK o o LNilContains}
            {RNilOK : NilOK ors ors RNilContains}
            (s_NilConsOK : ConsOK or s_RNilContains RNilContains s)
            (r_s_NilConsOK : ConsOK o r_s_RNilContains s_RNilContains r)
            (Nil_r_s_NilAppendOK : appendOK [] (r :> (s :> [])) contains)
            (s'_vsConsOK : ConsOK our s'_vsContains vsContains s')
            (r'_s'_vsConsOK : ConsOK ou r'_s'_vsContains s'_vsContains r')
            (us_r'_s'_vsAppendOK : appendOK us (r' :> (s' :> vs)) contains)
            ,
    («[] :+> r :> s :> []» <~~>*
     «us :+> r' :> s' :> vs»)
 → (ou = o) ∧ (ours = ors).
```

```
Proof with auto.
intros.
(*
dependent induction H
  generalizing
  or r s
  s_RNilContains r_s_RNilContains
  s_NilConsOK r_s_NilConsOK Nil_r_s_NilAppendOK.

    (*  This is the case where the transitive commute is finished *)
    destruct us.
        (*  This is the case where the original us is  *)
        split.
            apply NameSetEquality.
            symmetry.
            apply (nilFromEqTo (NilOK := nilOK)).
        dependent destruction x...
    simpl in x.
    destruct us.
        (*  This is the case where the original us is _ *)
        simpl in x.
        discriminate x.
    simpl in x.
    destruct us.
        (*  This is the case where the original us is _, _ *)
        simpl in x.
        discriminate x.
    simpl in x.
    (*  This is the case where the original us is (_:_:_:_) *)
    discriminate x.

(*  This is the case where we do a step of the transitive commute *)
simpl in H.
simpl in IHTransitiveCommuteRelates.
dependent destruction H.
    specialize (IHTransitiveCommuteRelates oq).
    specialize (IHTransitiveCommuteRelates q').
    specialize (IHTransitiveCommuteRelates p').

    specialize (IHTransitiveCommuteRelates pRsContains).
    specialize (IHTransitiveCommuteRelates contains).
    specialize (IHTransitiveCommuteRelates pRsConsOK).
    specialize (IHTransitiveCommuteRelates qPRsConsOK).
    solveFirstIn IHTransitiveCommuteRelates.
        constructor.
            remember (nilContainsNothing (NilOK := LNilOK)) as H1.
            clear - H1.
            signedNameSetDec.
        remember (nilContainsNothing (NilOK := LNilOK)) as H1.
        clear - H1.
        signedNameSetDec.
    solveFirstIn IHTransitiveCommuteRelates.
```

```
        f_equal.
        apply proof_irrelevance.
    solveFirstIn IHTransitiveCommuteRelates.
        f_equal.
    apply IHTransitiveCommuteRelates.

dependent destruction H.
dependent destruction H.
*)
admit.
Qed.

(*  XXX Can we merge this lemma with the previous one now that
   we don't require that Nil has from = to?
*)
Lemma TwoSequenceTransitiveCommuteRelatesTwoSequence2 :
    ∀ {pu_type : NameSet → NameSet → Type}
            {ppu : PartPatchUniverse pu_type pu_type}
            {pui : PatchUniverseInv ppu ppu}
            {patchUniverse : PatchUniverse pui}
            {o or ors our: NameSet}
            {usContains vsContains : SignedNameSet}
            {r : pu_type o or}
            {s : pu_type or ors}
            {us : Sequence patchUniverse usContains o o}
            {r' : pu_type o our}
            {s' : pu_type our ors}
            {vs : Sequence patchUniverse vsContains ors ors}

            {RNilContains s_RNilContains r_s_RNilContains : SignedNameSet}
            {LNilContains : SignedNameSet}
            {s'_vsContains r'_s'_vsContains : SignedNameSet}
            {contains : SignedNameSet}

            {LNilOK : NilOK o o LNilContains}
            {RNilOK : NilOK ors ors RNilContains}
            (s_NilConsOK : ConsOK or s_RNilContains RNilContains s)
            (r_s_NilConsOK : ConsOK o r_s_RNilContains s_RNilContains r)
            (Nil_r_s_NilAppendOK : appendOK [] (r :> (s :> [])) contains)
            (s'_vsConsOK : ConsOK our s'_vsContains vsContains s')
            (r'_s'_vsConsOK : ConsOK o r'_s'_vsContains s'_vsContains r')
            (us_r'_s'_vsAppendOK : appendOK us (r' :> (s' :> vs)) contains)
            ,

    («[] :+> r :> s :> []» <~~>* «us :+> r' :> s' :> vs»)
 → ∃ usNilOK : NilOK o o usContains,
    ∃ vsNilOK : NilOK ors ors vsContains,
    ((us = []) ∧ (vs = [])).
Proof with auto.
intros.
dependent induction H.
    dependent destruction us.
        ∃ nilOK.
```

```
        ∃ RNilOK.
        split...
    dependent destruction us.
    dependent destruction us.
simpl in H.
dependent destruction H.
    rename IHTransitiveCommuteRelates into IH.
    specialize (IH oq).
    specialize (IH our).
    specialize (IH usContains).
    specialize (IH vsContains).
    specialize (IH q').
    specialize (IH p').
    specialize (IH us).
    specialize (IH r').
    specialize (IH s').
    specialize (IH vs).
    specialize (IH RNilContains).
    specialize (IH pRsContains).
    specialize (IH contains).
    specialize (IH LNilContains).
    specialize (IH s'_vsContains).
    specialize (IH r'_s'_vsContains).
    specialize (IH LNilOK).
    specialize (IH RNilOK).
    specialize (IH pRsConsOK).
    specialize (IH qPRsConsOK).
    solveFirstIn IH.
        constructor.
            remember (nilContainsNothing (NilOK := LNilOK)) as H1.
            clear - H1.
            signedNameSetDec.
        remember (nilContainsNothing (NilOK := LNilOK)) as H1.
        clear - H1.
        signedNameSetDec.
    specialize (IH s'_vsConsOK).
    specialize (IH r'_s'_vsConsOK).
    specialize (IH us_r'_s'_vsAppendOK).
    solveFirstIn IH.
        simpl.
        f_equal.
        apply proof_irrelevance.
    solveFirstIn IH.
        f_equal.
    apply IH.
dependent destruction H.
dependent destruction H.
Qed.

Lemma EmptyNotCounterExample :
    ∀ {pu_type : NameSet → NameSet → Type}
        {ppu : PartPatchUniverse pu_type pu_type}
        {pui : PatchUniverseInv ppu ppu}
```

```
                {patchUniverse : PatchUniverse pui}
                {o op opq ou our ours : NameSet}
                {usContains vsContains : SignedNameSet}
                {p : pu_type o op}
                {q : pu_type op opq}
                {us : Sequence patchUniverse usContains o ou}
                {r : pu_type ou our}
                {s : pu_type our ours}
                {vs : Sequence patchUniverse vsContains ours opq}

                {RNilContains : SignedNameSet}
                {LNilContains : SignedNameSet}
                {q_RNilContains p_q_RNilContains : SignedNameSet}
                {s_vsContains r_s_vsContains : SignedNameSet}
                {contains : SignedNameSet}

                {LNilOK : NilOK o o LNilContains}
                {RNilOK : NilOK opq opq RNilContains}
                (q_NilConsOK : ConsOK op q_RNilContains RNilContains q)
                (p_q_NilConsOK : ConsOK o p_q_RNilContains q_RNilContains p)
                (Nil_p_q_NilAppendOK : appendOK [] (p :> (q :> [])) contains)
                (s_vsConsOK : ConsOK our s_vsContains vsContains s)
                (r_s_vsConsOK : ConsOK ou r_s_vsContains s_vsContains r)
                (us_r_s_vsAppendOK : appendOK us (r :> (s :> vs)) contains)
                ,
    TransitiveCommuteRelates ([] :+> p :> q :> [])
                             (us :+> r :> s :> vs)
 → (p <~?~> q)
 → (r <~?~> s).
Proof with auto.
intros.
assert (H3 := TwoSequenceTransitiveCommuteRelatesTwoSequence1 _ _ _ _ _ _ H).
destruct H3.
subst.
assert (H3 := TwoSequenceTransitiveCommuteRelatesTwoSequence2 _ _ _ _ _ _ H).
destruct H3 as [? [? [? ?]]].
subst.

dependent induction H...
dependent destruction H.
    simpl in IHTransitiveCommuteRelates.
    rename IHTransitiveCommuteRelates into IH.

    specialize (IH r_s_vsContains).
    specialize (IH s_vsContains).
    specialize (IH contains).
    specialize (IH pRsContains).
    specialize (IH LNilContains).
    specialize (IH LNilOK).
    specialize (IH RNilContains).
    specialize (IH RNilOK).
    specialize (IH vsContains).
    specialize (IH x0).
```

```
        specialize (IH usContains).
        specialize (IH x).
        specialize (IH our).
        specialize (IH s).
        specialize (IH s_vsConsOK).
        specialize (IH r).
        specialize (IH r_s_vsConsOK).
        specialize (IH us_r_s_vsAppendOK).
        specialize (IH oq).
        specialize (IH q').
        specialize (IH p').
        specialize (IH pRsConsOK).
        specialize (IH qPRsConsOK).
        solveFirstIn IH.
            constructor.
                remember (nilContainsNothing (NilOK := LNilOK)) as HN.
                clear - HN.
                signedNameSetDec.
            remember (nilContainsNothing (NilOK := LNilOK)) as HN.
            clear - HN.
            signedNameSetDec.
        solveFirstIn IH.
            apply commuteSelfInverse in H.
            unfold commutable.
            ∃ op.
            ∃ p.
            ∃ q...
        solveFirstIn IH.
            f_equal.
            apply proof_irrelevance.
        solveFirstIn IH.
            f_equal.
        apply IH.
    dependent destruction H.
    dependent destruction H.
    Qed.

    Lemma commuteInSequenceConsistent :
        ∀ {pu_type : NameSet → NameSet → Type}
            {ppu : PartPatchUniverse pu_type pu_type}
            {pui : PatchUniverseInv ppu ppu}
            {patchUniverse : PatchUniverse pui}
            {o oq oqr oqrs (*  oqrst *) ou our ours : NameSet}
            {qsContains : SignedNameSet}
            {qs : Sequence patchUniverse qsContains o oq}
            {r : pu_type oq oqr}
            {s : pu_type oqr oqrs}
        (*  {ts : Sequence patchUniverse _ oqrs oqrst} *)
            {usContains vsContains : SignedNameSet}
            {us : Sequence patchUniverse usContains o ou}
            {r' : pu_type ou our}
            {s' : pu_type our ours}
            {vs : Sequence patchUniverse vsContains ours oqrs}
```

```
                {NilContains s_NilContains r_s_NilContains : SignedNameSet}
                {s'_vsContains r'_s'_vsContains : SignedNameSet}
                {contains : SignedNameSet}

                (nilOK : NilOK oqrs oqrs NilContains)
                (s_NilConsOK : ConsOK oqr s_NilContains NilContains s)
                (r_s_NilConsOK : ConsOK oq r_s_NilContains s_NilContains r)
                (qs_r_s_NilAppendOK : appendOK qs (r :> s :> []) contains)

                (s'_vsConsOK : ConsOK our s'_vsContains vsContains s')
                (r'_s'_vsConsOK : ConsOK ou r'_s'_vsContains s'_vsContains r')
                (us_r'_s'_vsAppendOK : appendOK us (r' :> (s' :> vs)) contains)

                (transitive_commute : «qs :+> r :> s :> []» <~~>* «us :+> r' :> s' :> vs»)
                (rs_same : pu_nameOf r = pu_nameOf r')
                (ss_same : pu_nameOf s = pu_nameOf s')
                (r_s_commutable : r <~?~> s),
        (r' <~?~> s').
    Proof with trivial.
    intros.
    case (commutable_dec r' s')...
    intro r'_s'_not_commutable.
    destruct (EmptyCounterExampleExists _ _ _ _ _ _ _ transitive_commute rs_same ss_same
    r_s_commutable r'_s'_not_commutable)
        as [? [? [? [? [? [? [? [? [r" [s" [? [? [? [? [? [? [HX [? [? not_commutable]]]]]]]]]]]]]]]]]].
    assert (HY : «[] :+> r :> s :> []» <~~>* «[] :+> r :> s :> []»).
        apply Same.
    destruct (TwoSequenceTransitiveCommuteRelatesTwoSequence1 _ _ _ _ _ _ HX).
    destruct (TwoSequenceTransitiveCommuteRelatesTwoSequence1 _ _ _ _ _ _ HY).
    subst.
    destruct (TwoSequenceTransitiveCommuteRelatesTwoSequence2 _ _ _ _ _ _ HX) as [? [? [?
    ?]]].
    destruct (TwoSequenceTransitiveCommuteRelatesTwoSequence2 _ _ _ _ _ _ HY) as [? [? [?
    ?]]].
    subst.
    assert (r" <~?~> s").
        apply SymmetricTransitiveCommute in HX.
        apply (TransitiveTransitiveCommute HX) in HY.
        dependent destruction HX...
        dependent destruction H1.
            unfold commutable.
            ∃ oq0.
            ∃ q'.
            ∃ p'...
        dependent destruction H1.
        dependent destruction H1.
    contradiction.
    Qed.
```

XXX Is this right? Looks wrong to me now:

Lemma 9.2 (patch-in-sequence-has-minimal-context)
If $\overline{p}\overline{q}\overline{r}\overline{s}\overline{t} \leftrightarrow^* \overline{u}\overline{q'}\overline{v}$ (where $n(q) = n(q')$) and $n(q) \notin N(\overline{u})$, then $q \underset{?}{\leftrightarrow} \overline{r}\overline{s}$.

Explanation
Within a sequence, a patch has a minimal context.

Proof
Suppose, for the purpose of contraction, that we have a counterexample. Given s, let us consider the right-most q for which the property doesn't hold. Then $N(\overline{r}) \subseteq N(\overline{u})$, so at some point during the $\leftrightarrow^*$ relation q commuted with each of them, and with s. Therefore, by Lemma 9.1, it can be commuted past them all. But we assumed that it could not. Contradiction. ∎

Lemma 9.3 (patch-in-universe-has-minimal-context)
If $\overline{p}\overline{s}\overline{q}\overline{r}\overline{s}\overline{t}$ and $\overline{u}\overline{q'}\overline{v}$, where $n(q) = n(q')$ and $n(q) \notin N(\overline{u})$, are in a patch universe then $\langle q, \overline{r}\overline{s}\rangle \leftrightarrow \langle \overline{r'}\overline{s'}, q''\rangle$.

Explanation
Within a patch universe, a patch has a minimal context.

Proof
When a patch is first recorded, Lemma 9.2 tells us that it has a minimal context.

Another patch being recorded or unrecorded trivially doesn't alter its minimal context, as both operations act on the end of the patch sequence.

Commute trivially doesn't affect the minimal context, as the sequence doesn't change.

When merging, the patch is either in the common prefix or one of the suffixes. If it is in the common prefix or the suffix that remains unchanged then the minimal context is unaffected for the same reason that it is when recording new patches. If it is in the other suffix, then the definition of $\oplus$ tells us that we can commute it to give us the case where it is in the first suffix, so this case is satisfied too. ∎

XXX This is one of the important theorems: That given a patch universe, we can get to the point that the merge algorithm starts at

Theorem 9.1 (patch-universe-provides-merge-input)
Suppose we have two patch sequences $\overline{p}$ and $\overline{q}$.

Then $\langle \overline{p}\rangle \leftrightarrow^* \langle \overline{r}\overline{s}\rangle$ and $\langle \overline{q}\rangle \leftrightarrow^* \langle \overline{r}\overline{t}\rangle$ where $N(\overline{s}) \cap N(\overline{t}) = \emptyset$.

Explanation
XXX

Proof
XXX ∎

XXX

Sequences will be considered equal if they are equal up to commutation; in particular, when we talk about something like $\overline{pq}$ we mean "a sequence $\overline{r}$ which, after some number of commutations, is equal to $\overline{pq}$".

```
End PATCH_UNIVERSES.
```

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printing <˜> $\leftrightsquigarrow$ printing <˜?˜> $\overset{?}{\leftrightsquigarrow}$ printing □ ϵ contexted_patches

10 Contexted patches

```
Module Export CONTEXTED_PATCHES.

Require Import Equality.
Require Import names.
Require Import patch_universes.
```

Definition 10.1 (contexted-patch)
For all patches $r \in \mathbf{P}$, $: r$ is a *contexted patch* of r. For all contexted patches $\overline{q} : r$ and patches $p \in \mathbf{P}$, $p\overline{q} : r$ is a *contexted patch* of r if and only if $\langle p, \overline{q}r \rangle \leftrightarrow$ fail and $\overline{q} \neq p^{-1}\overline{q'}$ for some sequence $\overline{q'}$.

Explanation
Sometimes we will want to keep a "reference" to a patch p. However, for a patch to be useful we have to know what context it should be applied in. The purpose of a contexted patch is to keep track of a patch, and the context in which it applies.

The patch sequence before the colon is the context; the patch after the colon is the patch that we are interested in.

```
Inductive ContextedPatch {pu_type : (NameSet → NameSet → Type)}
                         {ppu : PartPatchUniverse pu_type pu_type}
                         {pui : PatchUniverseInv ppu ppu}
                         (pu : PatchUniverse pui)
                         (from : NameSet)
                       : Type
  := MkContextedPatch : ∀ {mid to : NameSet}
                          {cContains : SignedNameSet}
                          (c : Sequence pu cContains from mid)
                          (p : pu_type mid to),
                        ContextedPatch pu from.

(*  XXX Describe this in the latex: *)
Definition contextedPatch_name {pu_type : (NameSet → NameSet → Type)}
                               {ppu : PartPatchUniverse pu_type pu_type}
                               {pui : PatchUniverseInv ppu ppu}
                               {pu : PatchUniverse pui}
                               {from : NameSet}
                               (c : ContextedPatch pu from) : SignedName
 := match c with
    | MkContextedPatch _ _ _ _ p ⇒ pu_nameOf p
    end.
```

Definition 10.2 (patch-commute-past)
Given a patch p and a contexted patch $\overline{q} : r$, we define $\rightarrow$, pronounced *commute past*, thus:

$\langle p, \overline{q} : r \rangle \rightarrow \left\langle \overline{q'} : r' \right\rangle$ if $(\langle p, \overline{q} \rangle \leftrightarrow \left\langle \overline{q'}, p' \right\rangle) \wedge (\langle p', r \rangle \leftrightarrow \langle r', p'' \rangle)$

$\langle p, \overline{q} : r \rangle \rightarrow$ fail otherwise

```
Axiom cheat : ∀ {a}, a.

Inductive CommutePast {pu_type : (NameSet → NameSet → Type)}
                      {ppu : PartPatchUniverse pu_type pu_type}
                      {pui : PatchUniverseInv ppu ppu}
```

```
                          {pu : PatchUniverse pui}
                        : ∀ {from mid : NameSet},
                          pu_type from mid
                        → ContextedPatch pu mid
                        → ContextedPatch pu from
                        → Prop
  := CommutePastNil :
       ∀ (fromNilContains midNilContains : SignedNameSet)
           (from mid mid' to : NameSet)
           (p : pu_type from mid)
           (ident : pu_type mid to)
           (ident' : pu_type from mid')
           (p' : pu_type mid' to)
           (H : «p, ident» <~> «ident', p'»)
           (fromNilOK : NilOK from from fromNilContains)
           (midNilOK : NilOK mid mid midNilContains),
         CommutePast p
                     (MkContextedPatch _ _ [] ident)
                     (MkContextedPatch _ _ [] ident')
  | CommutePastCons :
       ∀ (rsContains qRsContains rs'Contains q'Rs'Contains : SignedNameSet)
           (o op opq opqr opqrs oq oqr oqrs : NameSet)
           (p : pu_type o op)
           (contextQ : pu_type op opq)
           (contextRs : Sequence pu rsContains opq opqr)
           (ident : pu_type opqr opqrs)

           (contextQ' : pu_type o oq)
           (contextRs' : Sequence pu rs'Contains oq oqr)
           (ident' : pu_type oqr oqrs)
           (p' : pu_type oq opq)

           (H : CommutePast p'
                (MkContextedPatch _ _ contextRs ident)
                (MkContextedPatch _ _ contextRs' ident'))

           (qRsConsOK : ConsOK op qRsContains rsContains contextQ)
           (q'Rs'ConsOK : ConsOK o q'Rs'Contains rs'Contains contextQ'),
         CommutePast p
                     (MkContextedPatch _ _ (contextQ :> contextRs) ident)
                     (MkContextedPatch _ _ (contextQ' :> contextRs') ident').

Definition CommutePastable {pu_type : (NameSet → NameSet → Type)}
                           {ppu : PartPatchUniverse pu_type pu_type}
                           {pui : PatchUniverseInv ppu ppu}
                           {pu : PatchUniverse pui}
                           {from mid : NameSet}
                           (p : pu_type from mid)
                           (cp : ContextedPatch pu mid)
                         : Prop
 := (∃ cp' : ContextedPatch pu from,
     CommutePast p cp cp').
```

```
Lemma CommutePastable_dec {pu_type : (NameSet → NameSet → Type)}
                          {ppu : PartPatchUniverse pu_type pu_type}
                          {pui : PatchUniverseInv ppu ppu}
                          {pu : PatchUniverse pui}
                          {from mid : NameSet}
                          (p : pu_type from mid)
                          (cp : ContextedPatch pu mid)
                        : {CommutePastable p cp} + {¬CommutePastable p cp}.
Proof.
intros.
destruct cp.
dependent induction c.
    destruct nilOK.
    remember (NameSetEquality nilFromEqTo0) as H.
    clear HeqH.
    subst.
    destruct (commutable_dec p p0).
        left.
        destruct c as [? [p0' [p' ?]]].
        assert (nilFromOK : NilOK from from SignedNameSetMod.empty).
            constructor.
                nameSetDec.
            signedNameSetDec.
        ∃ (MkContextedPatch pu from [] p0').
        (*  XXX Tidy this up: *)
        apply (CommutePastNil
                   SignedNameSetMod.empty
                   contains
                   from
                   to0
                   x
                   to
                   p
                   p0
                   p0'
                   p'
                   H
                   _
                   _
                  ).
    right.
    admit. (*  XXX *)
admit. (*  XXX *)
Qed.
Inductive CommuteManyPast {pu_type : (NameSet → NameSet → Type)}
                          {ppu : PartPatchUniverse pu_type pu_type}
                          {pui : PatchUniverseInv ppu ppu}
                          {pu : PatchUniverse pui}
                        : ∀ {contains : SignedNameSet}
                                    {from mid : NameSet},
                            Sequence pu contains from mid
                          → ContextedPatch pu mid
```

```
                          → ContextedPatch pu from
                          → Prop
    := CommuteNilPast : ∀ (fromNilContains : SignedNameSet)
                                (from : NameSet)
                                (cp : ContextedPatch pu from)
                                (fromNilOK : NilOK from from fromNilContains),
                          CommuteManyPast [] cp cp
     | CommuteConsPast : ∀ (qsContains pQsContains : SignedNameSet)
                                (o op opq : NameSet)
                                (p : pu_type o op)
                                (qs : Sequence pu qsContains op opq)
                                (cp : ContextedPatch pu opq)
                                (cp' : ContextedPatch pu op)
                                (cp'' : ContextedPatch pu o)
                                (CommuteQsPast : CommuteManyPast qs cp cp')
                                (CommutePPast : CommutePast p cp' cp'')
                                (consOK : ConsOK o pQsContains qsContains p),
                          CommuteManyPast (p :> qs) cp cp''.
```

```
(*
Definition commutePast {pu : PatchUniverse}
                       {from mid : NameSet}
                       (p : (pu_type pu) from mid)
                       (c : ContextedPatch pu mid)
                     : option (ContextedPatch pu from)
 := match c with
    (*  XXX Should have Nil = cContext *)
    | MkContextedPatch _ _ _ (Nil _) cPatch =>
          match (commutable_dec pu) p (cheat cPatch) with
          | left _ => None
          | right _ => None
          end
    | _ => None
    end.
*)
```

Definition 10.3 (patch-commute-past-set)
We extend $\rightarrow$ to work on sets of contexted patches in the natural way, i.e. for any patch p and set of contexted patches S:
$\langle p, S\rangle \rightarrow \left\langle \left\{ (\overline{q}' : r') \mid (\overline{q} : r) \in S \wedge \langle p, \overline{q} : r\rangle \rightarrow \left\langle \overline{q}' : r' \right\rangle \right\} \right\rangle$
if all the commute pasts succeed, and $\langle p, S\rangle \rightarrow$ fail otherwise.

From here on we assume that contexted patches magically maintain their invariant, i.e. if we have a patch p and a contexted patch $\overline{q} : r$ then when we write $p\overline{q} : r$ what we mean is, if $\langle p, \overline{q} : r\rangle \rightarrow \left\langle \overline{q}' : r' \right\rangle$ then $\overline{q}' : r'$, otherwise $p\overline{q} : r$.

```
Definition addToContext {pu_type : (NameSet → NameSet → Type)}
                        {ppu : PartPatchUniverse pu_type pu_type}
                        {pui : PatchUniverseInv ppu ppu}
                        {pu : PatchUniverse pui}
                        {from mid : NameSet}
                        (p : pu_type from mid)
```

```
                          (c : ContextedPatch pu mid)
                        : ContextedPatch pu from
 := cheat.
(*
match c with
    | MkContextedPatch _ _ _ _ p => pu_nameOf _ p
    end.
*)
Program Fixpoint addSequenceToContext
                        {pu_type : (NameSet → NameSet → Type)}
                        {ppu : PartPatchUniverse pu_type pu_type}
                        {pui : PatchUniverseInv ppu ppu}
                        {pu : PatchUniverse pui}
                        {contains : SignedNameSet}
                        {from mid : NameSet}
                        (ps : Sequence pu contains from mid)
                        (c : ContextedPatch pu mid)
                      : ContextedPatch pu from
 := match ps with
    | Nil _ ⇒ c
    | Cons _ _ _ p ps' _ ⇒
          (addToContext p (addSequenceToContext ps' c))
    end.
Next Obligation.
symmetry.
apply NameSetEquality.
apply nilFromEqTo.
Qed.
Next Obligation.
apply NameSetEquality.
apply consFromOK.
Qed.
```

Definition 10.4 (contexted-patch-conflict)
We define $\leftrightarrows$, pronounced "does not conflict with", such that $(\overline{p} : q) \leftrightarrows (\overline{r} : s)$ holds if $\langle q^{-1}, \overline{p}^{-1}\overline{r} : s\rangle \to \langle \_ \rangle$, and does not hold otherwise.

Explanation
Here $\overline{p} : q$ and $\overline{r} : s$ are two contexted patches starting from the same context. By inverting one of them we can put them into a single patch sequence $q^{-1}\overline{p}^{-1}\overline{r}s$. By building the contexted patch $\overline{p}^{-1}\overline{r} : s$ if $\overline{p}$ and $\overline{r}$ both contain a patch t, that patch will be removed (as contexted patches magically maintain their invariant).

This is almost, but not quite the same thing as saying $\overline{p}q$ and $\overline{r}s$ can be cleanly merged. For a counter-example, consider $: t$ and $t : u$. Clearly t and tu can be cleanly merged, giving tu, but $\langle t^{-1}, t : u\rangle \to$ fail. Likewise, starting with the contexted patches the other way round, we get $\langle u^{-1}, t^{-1} : t\rangle \to$ fail.

```
Definition invertContextedPatch {pu_type : (NameSet → NameSet → Type)}
                                {ppu : PartPatchUniverse pu_type pu_type}
                                {pui : PatchUniverseInv ppu ppu}
```

```
                                    {pu : PatchUniverse pui}
                                    {from : NameSet}
                                    (c : ContextedPatch pu from)
                                  : ContextedPatch pu from
 := cheat.
(*
 := match c with
    | MkContextedPatch _ _ _ ps ident =>
          addSequenceToContext ps
              (addToContext ident
                   (MkContextedPatch _ _  (invert _ ident)))
    end.
*)
End CONTEXTED_PATCHES.
```

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printing $<\tilde{}>$ $\leftrightsquigarrow$ printing $<\tilde{}?\tilde{}>$ $\overset{?}{\leftrightsquigarrow}$ printing $\square$ ϵ catches_definition

11 Catches and Repos

```
Module Export catches_definition.

Require Import Equality.
Require Import List.

Require Import names.
Require Import patch_universes.
Require Import patch_universes_sequences.
Require Import invertible_patchlike.
Require Import contexted_patches.

(*  XXX Move this *)
Lemma map_neq_nil : ∀ {t1 t2 : Type}
                              {l : list t1}
                              {f : t1 → t2},
                       l ≠ nil
                    → map f l ≠ nil.
Proof with auto.
intros.
intro.
apply map_eq_nil in H0.
congruence.
Qed.

(*  XXX Move this *)
Lemma map_neq_nil2 : ∀ {t1 t2 : Type}
                              {l : list t1}
                              {f : t1 → t2},
                       map f l ≠ nil
                    → l ≠ nil.
```

```
Proof with auto.
intros.
intro.
elim H.
subst.
unfold map...
Qed.
```

Now that we have laid the foundations, it is time to introduce catches. We will now be dealing with another datatype, which may either be a patch (as previously defined) or a *conflictor*. We will call these beasts *catches*.

Explanation
The basic idea is that, if two catches do not conflict, then we can merge them, similar to the way that we merge patches. However, if they do conflict then when we merge, we get a conflictor which records the conflict. To compute the contents of a repository we take the effects of all the patches that are in the repository and do not conflict with any other patch in the repository.

Definition 11.1 (catches)
A catch is either $[p]$ (the non-conflicted patch $p \in \mathbf{P}$), $\left[\overline{r}, X, \overline{p} : q\right]$ (a conflicted patch $q \in \mathbf{P}$), or $\left[\!\left[\overline{r}, X, \overline{p} : q\right]\!\right]$ (the inverse of a conflicted patch $q \in \mathbf{P}$). In both cases, $\overline{r}$ is a sequence of patches, X is a set of contexted patches, and $\overline{p} : q$ is a contexted patch.

```
Inductive Catch {pu_type : NameSet → NameSet → Type}
                {ppu : PartPatchUniverse pu_type pu_type}
                {pui : PatchUniverseInv ppu ppu}
                {pu : PatchUniverse pui}
                (ipl : InvertiblePatchlike pu)
                (from to : NameSet)
              : Type
    := MkCatch : ∀ (p : pu_type from to),
                 Catch ipl from to
     | Conflictor : ∀ {effectContains : SignedNameSet}
                      (effect : Sequence pu effectContains from to)
                      (conflicts : list (ContextedPatch pu to))
                      (ident : ContextedPatch pu to),
                      (*  XXX Proof conflicts /= ?
                              Proof effect^ \subseteq conflicts?
                              Proof conflicts no dupes? *)
                    Catch ipl from to.
Implicit Arguments MkCatch [pu_type ppu pui pu ipl from to].
Implicit Arguments Conflictor [pu_type ppu pui pu ipl from to effectContains].
```

Definition 11.2 (repos)
A *repo* is a sequence of catches (up to commutation, to be defined later) satisfying some requirements (that, again, we will give later).

Let $\mathbf{R}$ be the (possibly infinite) set of repos.

Aside
Do we really need inverse conflictors, or can we just use conflictors and invert their internals?

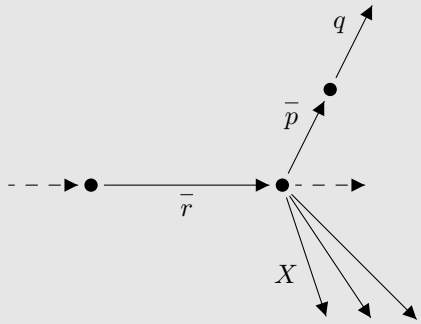
The meaning of $[p]$ is hopefully clear, but what is the meaning of $\left[\overline{r}, X, \overline{p} : q\right]$? To answer this question, we need to consider it in the context of a repo.

Suppose we have the repo $\overline{c}\left[\overline{r}, X, \overline{p} : q\right]$. The effect of the conflictor on the repo is $\overline{r}$, and as we have already said, $\overline{p} : q$ is the (contexted) patch that this conflictor represents. X is the set of (contexted) patches in $\overline{c}$ that q conflicts with.

Aside
In darcs' theory, the transitive set of conflicts is stored. I don't believe that this is needed, and not having it makes things simpler. Not having it may also mean more things commute.

But how is $\overline{r}$ calculated? $\overline{r}$ is the inverses of the subset of the patches in X that do not appear in the effect of any conflictor in $\overline{c}$. In other words, the first catch to conflict with any given patch reverts that patch.

We can picture a conflictor $\left[\overline{r}, X, \overline{p} : q\right]$ in a repo as looking like this:



Definition 11.3 (catch-effect)
We define $\mathscr{E}$ to tell us the effect that a catch has on the repo:
$\mathscr{E}\left([p]\right) = p$
$\mathscr{E}\left(\left[\overline{r}, X, y\right]\right) = \overline{r}$
$\mathscr{E}\left(\left[\!\left[\overline{r}, X, y\right]\!\right]\right) = \overline{r}^{-1}$

Definition 11.4 (catch-conflicts)
We define $\mathscr{C}$ to tell us the patches that a catch conflicts with, i.e.:
$\mathscr{C}\left([p]\right) = \emptyset$
$\mathscr{C}\left(\left[\overline{r}, X, y\right]\right) = N\left(X\right)$
$\mathscr{C}\left(\left[\!\left[\overline{r}, X, y\right]\!\right]\right) = N\left(X\right)^{-1}$

Definition 11.5 (catch-names)
We extend n and N to work on catches in the natural way, i.e.:
$n\left([p]\right) = p$
$n\left(\left[\overline{r}, X, \overline{p} : q\right]\right) = n\left(q\right)$
$n\left(\left[\!\left[\overline{r}, X, \overline{p} : q\right]\!\right]\right) = n\left(q\right)^{-1}$
$N\left(\epsilon\right) = \emptyset$
$N\left(c\overline{d}\right) = \left\{n\left(c\right)\right\} \cup N\left(\overline{d}\right)$

```
Definition catch_name {pu_type : NameSet → NameSet → Type}
                      {ppu : PartPatchUniverse pu_type pu_type}
                      {pui : PatchUniverseInv ppu ppu}
                      {pu : PatchUniverse pui}
                      {ipl : InvertiblePatchlike pu}
                      {from to : NameSet}
```

```
                                (c : Catch ipl from to) : SignedName
 := match c with
    | MkCatch p ⇒ pu_nameOf p
    | Conflictor _ _ _ p ⇒ contextedPatch_name p
    end.
```

Definition 11.6 (repo-properties)
Repos are inductively defined as follows:

ϵ is a repo.

$\overline{c}\,[p]$ is a repo if

- $\overline{c}$ is a repo
- $n\,(p)$ is positive
- $n\,(p) \notin N\left(\overline{c}\right)$
- $\mathscr{E}\,(c)\,p$ is sensible

$\overline{cde}\left[\overline{r}, X, \overline{p} : q\right]$ is a repo if:

- $\overline{cde}$ is a repo
- $\overline{e}$ is a sequence of patches (i.e. there are no conflictors in $\overline{e}$)
- $\mathscr{E}\left(\overline{e}\right) = \overline{r}^{-1}$
- $\mathscr{E}\left(\overline{d}\right) = \overline{p}^{-1}$
- $N\left(\overline{de}\right) = N\,(X)$
- Every catch in d is either a conflictor, or in $\mathscr{C}\,(d)$
- $n\,(y)$ is positive
- $n\,(y) \notin N\left(\overline{cde}\right)$
- There is no catch f and catch sequence $\overline{g}$ such that $\overline{de} = f\overline{g}$ and f does not conflict with $[q]$.
- $\mathscr{E}\,(c)\,q$ is sensible

Definition 11.7 (catch-inverse)
We define
$[p]^{-1} = \left[p^{-1}\right]$
$\left[\overline{r}, X, y\right]^{-1} = \left[\!\left[\overline{r}, X, y\right]\!\right]$
$\left[\!\left[\overline{r}, X, y\right]\!\right]^{-1} = \left[\overline{r}, X, y\right]$

```
(*  XXX Put this elsewhere? *)
Definition invertSignedNameSet (s : SignedNameSet)
                               : SignedNameSet
 := SignedNameSetMod.fold (fun sn s' ⇒ SignedNameSetAdd (signedNameInverse sn) s') s
SignedNameSetMod.empty.

(*  XXX Put this elsewhere? *)
Program Fixpoint invertSequence {pu_type : NameSet → NameSet → Type}
                                {ppu : PartPatchUniverse pu_type pu_type}
                                {pui : PatchUniverseInv ppu ppu}
                                {pu : PatchUniverse pui}
```

```
                                    {ipl : InvertiblePatchlike pu}
                                    {from to : NameSet}
                                    {psContains : SignedNameSet}
(*
XXX
                                      (nilOK : NilOK from from SignedNameSetMod.empty)
*)
                                    (ps : Sequence pu psContains from to)
                                  : Sequence pu (invertSignedNameSet psContains) to from
 := match ps with
    | Nil _ ⇒ []
    | Cons _ _ _ p ps' _ ⇒ invertSequence ps' :+>
                                 ((invert p :> Nil _ (contains := SignedNameSetMod.empty))
                                                      : Sequence pu
                                                        (SignedNameSetAdd    (signedNameIn-
verse (pu_nameOf p)) SignedNameSetMod.empty)
                                                        _
                                                        from)
    end.
Next Obligation.
destruct wildcard'.
split.
    nameSetDec.
admit.
Qed.
Next Obligation.
constructor.
    destruct wildcard'2.
    auto.
signedNameSetDec.
Qed.
Next Obligation.
split.
        nameSetDec.
    nameSetDec.
admit. (*  XXX Needs an extra proof req in patch universes,
              that name of inverse is inverse of name? *)
Qed.
Next Obligation.
    split.
        signedNameSetDec.
    admit.
split.
    admit.
admit.
Defined.

(*
Definition CatchInverse {ipl : InvertiblePatchlike}
                        {from to : NameSet}
                        (c : Catch ipl from to)
                      : Catch ipl to from
 := match c with
```

```
      | MkCatch p => MkCatch (invert _ p)
      | Conflictor _ effect conflicts ident =>
           Conflictor (invertSequence effect)
                      (map (fun cp => addSequenceToContext effect (invertContextedPatch cp)) co
                      (addSequenceToContext effect (invertContextedPatch ident))
    end.
*)
```

11.1 Merge

We now define merge for catches.

Definition 11.8 (merge)
We write the merge operator for catches as $+$. It cannot fail: It introduces conflictors instead.
$\forall (\overline{c}d) \in \mathbf{R}, (\overline{c}e) \in \mathbf{R} \cdot$
$(n\,(d) = n\,(e)) \vee$
$\exists d' \in \mathbf{C}, e' \in \mathbf{C} \cdot$
$d + e = \langle e', d' \rangle$

Explanation
What we're saying here is that if we have two repos that differ in only their last patch then we can always merge them. The two (equal up to commutation) results of the repo merge are $\overline{c}de'$ and $\overline{c}ed'$.

We define $+$ thus:

$$
\begin{array}{lll}
c & +d & = \langle d', c' \rangle \\
 & & \quad \textbf{if } \left\langle c^{-1}, d \right\rangle \leftrightarrow \left\langle d', c'^{-1} \right\rangle \\
[p] & +[q] & = \left\langle \left[p^{-1}, \{:p\}, :q\right], \left[q^{-1}, \{:q\}, :p\right] \right\rangle \\
[p] & +\left[\overline{r}, X, y\right] & = \left\langle \left[p^{-1}\overline{r}, \left\{\overline{r}^{-1} : p\right\} \cup X, y\right], \left[\epsilon, \{y\}, \overline{r}^{-1} : p\right] \right\rangle \\
\left[\overline{r}, X, y\right] & +[q] & = \left\langle \left[\epsilon, \{y\}, \overline{r}^{-1} : q\right], \left[q^{-1}\overline{r}, \left\{\overline{r}^{-1} : q\right\} \cup X, y\right] \right\rangle \\
\left[\overline{rs}, W, x\right] & +\left[\overline{rt}, Y, z\right] & = \left\langle \left[\overline{t}', \left\{\overline{t}'^{-1}x\right\} \cup (\overline{s}'^{-1}Y), \overline{s}'^{-1}z\right], \left[\overline{s}', \left\{\overline{s}'^{-1}z\right\} \cup (\overline{t}'^{-1}W), \overline{t}'^{-1}x\right] \right\rangle \\
 & & \quad \textbf{if } N\left(\overline{s}\right) \cap N\left(\overline{t}\right) = \emptyset \\
 & & \quad \left\langle \overline{s}^{-1}, \overline{t} \right\rangle \leftrightarrow \left\langle \overline{t}', \overline{s}'^{-1} \right\rangle
\end{array}
$$

Note that we haven't yet defined $\leftrightarrow$ on catches, but based on how it works on patches you should have some intuition for what it means.

Explanation
The first rule handles the case where there is no conflict. This works just like patch merging.

The second rule handles the case where we have two patches, p and q, that conflict. If we have p in our repo and we pull q, then we make a conflictor that inverts p, conflicts with p, and represents q.

Pulling p into a repo containing q is analogous.

The third rule handles the case where we have a patch $[p]$ in one repository and a conflictor, $\left[\overline{r}, X, y\right]$ in the other repository, and they conflict.

If we pull the conflictor into the repository containing the patch then the conflictor is the first catch to conflict with p, so it must revert it (along with everything it already reverts). It also needs to record that it conflicts with p, along with everything that it conflicted with before. And finally, it records that its identity is still y.

On the other hand, if we pull the patch into a repo containing the conflictor, then the patch turns into a conflictor too. It doesn't need to revert y as it is not the first catch to conflict with it: everything in X has already conflicted with it (XXX lemma that X is not empty). Therefore it has no effect. It does need to record that it conflicts with y (and only y). And, of course, it records that its identity is p.

The fourth rule is the same as the third rule, but with the catches in the opposite order.

The fifth rule handles the case where we have a conflictor in each repo, and the conflictors also conflict with each other. The mechanics of the rule are similar to the previous two rules, but we need to do some setup work before we can apply it. The x conflictor is the first in its repo to conflict with everything in $\overline{rs}$, and the z conflictor is the first in its repo to conflict with everything in $\overline{rt}$. But if we pull the z into the repo already containing y then the z conflictor won't be the first catch to conflict with $\overline{r}$, as x already conflicts with it. So we need to commute all common patches in the two conflictor effects to the left. Furthermore, the details of the rule requires that the remaining effects do not conflict with each other. As merging cannot fail, we will have to show that this is the case!

11.2 Commute

We now define commutation of catches.

Definition 11.9 (catch-commute)
We extend $\leftrightarrow$ to operate on catches, as defined below.

We can break the rules down into 3 classes: Those where the patches are the result of a conflicting merge, and the conflicts gets reshuffled when they commute; those where the patches are unrelated, and simply commute freely; and a fail case for everything else.

Currently we don't give the commute rules for anything involving inverse conflictors. We don't believe that those rules will raise any new problems, though.

Conflicted merge

First, if we have a patch, and a conflictor that has only conflicted with that patch, then they swap places:
$\langle [p], [p^{-1}, \{:p\}, :q] \rangle \leftrightarrow \langle [q], [q^{-1}, \{:q\}, :p] \rangle$

Explanation
This should make sense if you consider the result of merging two patches:
$[p] + [q] = \langle [p^{-1}, \{:p\}, :q], [q^{-1}, \{:q\}, :p] \rangle$

If we have two conflictors, but the one on the right only conflicts with the one on the left, then it becomes a patch after it has commuted:
$\langle [\overline{r}, X, y], [\epsilon, \{y\}, \overline{r}^{-1} : q] \rangle \leftrightarrow \langle [q], [q^{-1}\overline{r}, \{\overline{r}^{-1} : q\} \cup X, y] \rangle$

Explanation
Again, this follows from the merge:
$[\overline{r}, X, y] + [q] = \langle [\epsilon, \{y\}, \overline{r}^{-1} : q], [q^{-1}\overline{r}, \{\overline{r}^{-1} : q\} \cup X, y] \rangle$

And the inverse of the previous case:
$\langle [p], [p^{-1}\overline{r}, \{\overline{r}^{-1} : p\} \cup X, y] \rangle \leftrightarrow \langle [\overline{r}, X, y], [\epsilon, \{y\}, \overline{r}^{-1} : p] \rangle$

Explanation
Again, this follows from the merge:
$[p] + [\overline{r}, X, y] = \langle [p^{-1}\overline{r}, \{\overline{r}^{-1} : p\} \cup X, y], [\epsilon, \{y\}, \overline{r}^{-1} : p] \rangle$

And now the case where both are conflictors, and conflict with each other:
$\left\langle \left[\overline{rs}, W, x\right], \left[\overline{t}, \left\{\overline{t}^{-1}x\right\} \cup Y, z\right]\right\rangle \leftrightarrow \left\langle \left[\overline{rt'}, \overline{s'}Y, \overline{s'}z\right], \left[\overline{s'}, \{z\} \cup \overline{t}^{-1}W, \overline{t}^{-1}x\right]\right\rangle$
if $\left\langle \overline{s}, \overline{t}\right\rangle \leftrightarrow \left\langle \overline{t'}, \overline{s'}\right\rangle$
$N\left(\overline{r}^{-1}\right) \subseteq N\left(Y\right)$
$N\left(\overline{s}^{-1}\right) \cap N\left(Y\right) = \emptyset$

Explanation
XXX Copy the merge rule and rewrite

In this case we just move the conflict from one to the other. The splitting of the effect of the z conflictor into two parts is the same as we saw earlier, in the "unrelated" conflict-conflictor commute.

Unrelated

First the simple non-conflicted patch case:
$\left\langle \left[p\right], \left[q\right]\right\rangle \leftrightarrow \left\langle \left[q'\right], \left[p'\right]\right\rangle$ if $\left\langle p, q\right\rangle \leftrightarrow \left\langle q', p'\right\rangle$

Explanation
If our catches just wrap up patches, then they commute like the underlying patches.

Now commute a conflictor past a patch, where everything goes smoothly:
$\left\langle \left[\overline{r}, X, y\right], \left[q\right]\right\rangle \leftrightarrow \left\langle \left[q'\right], \left[\overline{r'}, X', y'\right]\right\rangle$ if $\left\langle \overline{r}, q\right\rangle \leftrightarrow \left\langle q', \overline{r'}\right\rangle$
$\left\langle q^{-1}, y\right\rangle \rightarrow \left\langle y'\right\rangle$
$\left\langle q^{-1}, X\right\rangle \rightarrow \left\langle X'\right\rangle$

Explanation
XXX explanation and pretty pictures will be later.

Next we have the case where the patch and conflictor start off the other way round:
$\left\langle \left[p\right], \left[\overline{r}, X, y\right]\right\rangle \leftrightarrow \left\langle \left[\overline{r'}, X', y'\right], \left[p'\right]\right\rangle$ if $\left\langle p, \overline{r}\right\rangle \leftrightarrow \left\langle \overline{r'}, p'\right\rangle$
$\left\langle p', X\right\rangle \rightarrow \left\langle X'\right\rangle$
$\left\langle p', y\right\rangle \rightarrow \left\langle y'\right\rangle$

Explanation
XXX ditto

And the last and most complex of the unrelated cases, commuting a conflictor past another conflictor:
$\left\langle \left[\overline{rs}, W, x\right], \left[\overline{t}, Y, z\right]\right\rangle \leftrightarrow \left\langle \left[\overline{rt'}, \overline{s'}Y, z'\right], \left[\overline{s'}, \overline{t}^{-1}W, x'\right]\right\rangle$
if $N\left(\overline{r}^{-1}\right) \subseteq N\left(Y\right)$
$N\left(\overline{s}^{-1}\right) \cap N\left(Y\right) = \emptyset$
$\left\langle \overline{s}, \overline{t}\right\rangle \leftrightarrow \left\langle \overline{t'}, \overline{s'}\right\rangle$
$x \leftrightarrows \overline{t}z$
$\forall w \in W \cdot (w \leftrightarrows \overline{t}z)$
$\forall y \in Y \cdot (x \leftrightarrows \overline{t}y)$
$\left\langle \overline{t}^{-1}, x\right\rangle \rightarrow \left\langle x'\right\rangle$
$\left\langle \overline{s'}, z\right\rangle \rightarrow \left\langle z'\right\rangle$

Explanation
XXX ditto

Fail

Finally, if none of the above hold, then
$\langle c, d\rangle \hookrightarrow$ fail

Definition 11.10 (catch-conflicts)
If $(\overline{c}d) \in \mathbf{R}$ and $(\overline{c}e) \in \mathbf{R}$, We say d *conflicts* with e if and only if $\left\langle d^{-1}, e\right\rangle \hookrightarrow$ fail.

Explanation
XXX

```
(*  XXX *)
Axiom cheat : ∀ {a}, a.
(*
(*  XXX Put these things below in the right places: *)
Inductive commuteOneMany : forall {pu_type : NameSet -> NameSet -> Type}
                                  {ppu : PartPatchUniverse pu_type pu_type}
                                  {pui : PatchUniverseInv ppu ppu}
                                  {pu : PatchUniverse pui}
                                  {from mid1 mid2 to : NameSet}
                                  {qsContains : SignedNameSet}
                                  (p   : pu_type from mid1)
                                  (qs  : Sequence pu  mid1 to   qsContains)
                                  (qs' : Sequence pu  from mid2 qsContains)
                                  (p'  : pu_type mid2 to),
                           Prop
    := commuteOneNil :
          forall {pu : PatchUniverse}
                 {o op : NameSet}
                 {nilContains : SignedNameSet}
                 (oNilOK : NilOK o o nilContains)
                 (opNilOK : NilOK op op nilContains)
                 (p : pu_type o op),
          commuteOneMany p   p
     | commuteOneCons :
          forall {pu : PatchUniverse}
                 {o op opq opqr oq oqr : NameSet}
                 {rsContains qRsContains : SignedNameSet}
                 (p : (pu_type pu) o op)
                 (p' : (pu_type pu) oq opq)
                 (p'' : (pu_type pu) oqr opqr)
                 (q : (pu_type pu) op opq)
                 (q' : (pu_type pu) o oq)
                 (rs : Sequence pu rsContains opq opqr)
                 (rs' : Sequence pu rsContains oq oqr)
                 (qRsConsOK : ConsOK op
                                     qRsContains
                                     rsContains
                                     q)
                 (q'Rs'ConsOK : ConsOK o
                                       qRsContains
                                       rsContains
```

```
                                    q')
                (pq_commute_q'p' : «p, q» <~> «q', p'»)
                (p'qs_commute_qs'p'' : commuteOneMany p' rs rs' p''),
          commuteOneMany p (q :> rs) (q' :> rs') p''.

Inductive commuteManyOne : forall {pu : PatchUniverse}
                                  {from mid1 mid2 to : NameSet}
                                  {psContains : SignedNameSet}
                                  (ps  : Sequence pu  from mid1  psContains)
                                  (q   : (pu_type pu) mid1 to)
                                  (q'  : (pu_type pu) from mid2)
                                  (ps' : Sequence pu  mid2 to    psContains),
                           Prop
    := commuteNilOne :
          forall {pu : PatchUniverse}
                 {o oq : NameSet}
                 {nilContains : SignedNameSet}
                 (nilOK : NilOK o o nilContains)
                 (nilOK : NilOK oq oq nilContains)
                 (q : (pu_type pu) o oq),
          commuteManyOne  q q
     | commuteConsOne :
          forall {pu : PatchUniverse}
                 {o op opq opqr or opr : NameSet}
                 {qsContains pQsContains : SignedNameSet}
                 (p : (pu_type pu) o op)
                 (p' : (pu_type pu) or opr)
                 (qs : Sequence pu qsContains op opq)
                 (qs' : Sequence pu qsContains opr opqr)
                 (r : (pu_type pu) opq opqr)
                 (r' : (pu_type pu) op opr)
                 (r'' : (pu_type pu) o or)
                 (pQsConsOK : ConsOK o
                                     pQsContains
                                     qsContains
                                     p)
                 (p'Qs'ConsOK : ConsOK or
                                       pQsContains
                                       qsContains
                                       p')
                 (qsr_commute_r'qs' : commuteManyOne qs r r' qs')
                 (pr'_commute_r''p' : «p, r'» <~> «r'', p'»),
          commuteManyOne (p :> qs) r r'' (p' :> qs').

Lemma commute_OneMany_ManyOne
    : forall {pu : PatchUniverse}
             {from mid1 mid2 to : NameSet}
             {qsContains : SignedNameSet}
             (p   : (pu_type pu) from mid1)
             (qs  : Sequence pu  mid1 to   qsContains)
             (qs' : Sequence pu  from mid2 qsContains)
             (p'  : (pu_type pu) mid2 to),
```

```
      commuteOneMany p qs qs' p'
   -> commuteManyOne qs' p' p qs.
Proof.
intros.
admit.
Qed.

Lemma commute_ManyOne_OneMany
    : forall {pu : PatchUniverse}
             {from mid1 mid2 to : NameSet}
             {psContains : SignedNameSet}
             (ps  : Sequence pu  from mid1  psContains)
             (q   : (pu_type pu) mid1 to)
             (q'  : (pu_type pu) from mid2)
             (ps' : Sequence pu  mid2 to    psContains),
      commuteManyOne ps q q' ps'
   -> commuteOneMany q' ps' ps q.
Proof.
intros.
admit.
Qed.
*)

Inductive AllCommutePast {pu_type : NameSet → NameSet → Type}
                         {ppu : PartPatchUniverse pu_type pu_type}
                         {pui : PatchUniverseInv ppu ppu}
                         {pu : PatchUniverse pui}
                         {ipl : InvertiblePatchlike pu}
                         {from mid : NameSet}
                         (p : pu_type from mid)
                       : list (ContextedPatch pu mid)
                       → list (ContextedPatch pu from)
                       → Prop
    := AllCommutePastNil :
         AllCommutePast p nil nil
     | AllCommutePastCons :
         ∀ {cp : ContextedPatch pu mid}
           {cps : list (ContextedPatch pu mid)}
           {cp' : ContextedPatch pu from}
           {cps' : list (ContextedPatch pu from)},
         CommutePast p cp cp' →
         AllCommutePast p cps cps' →
         AllCommutePast p (cons cp cps) (cons cp' cps').

(*  XXX Want to be able to use
          «ps, qs» <~> «qs', ps'»
       for this, but it doesn't work currently *)
Parameter sequenceCommute : ∀ {pu_type : NameSet → NameSet → Type}
                                 {ppu : PartPatchUniverse pu_type pu_type}
                                 {pui : PatchUniverseInv ppu ppu}
                                 {pu : PatchUniverse pui}
                                 {ipl : InvertiblePatchlike pu}
                                 {from mid mid' to : NameSet}
```

```
                                        {psContains qsContains ps'Contains qs'Contains :
SignedNameSet },
                                  Sequence pu psContains from mid →
                                  Sequence pu qsContains mid to →
                                  Sequence pu qs'Contains from mid' →
                                  Sequence pu ps'Contains mid' to →
                                  Prop.
Parameter sequenceCommuteInverse
    : ∀ {pu_type : NameSet → NameSet → Type}
                {ppu : PartPatchUniverse pu_type pu_type}
                {pui : PatchUniverseInv ppu ppu}
                {pu : PatchUniverse pui}
                {ipl : InvertiblePatchlike pu}
                {from mid mid' to : NameSet}
                {psContains qsContains ps'Contains qs'Contains : SignedNameSet}
                (ps : Sequence pu psContains from mid )
                (qs : Sequence pu qsContains mid to )
                (qs' : Sequence pu qs'Contains from mid')
                (ps' : Sequence pu ps'Contains mid' to ),
       sequenceCommute ps qs qs' ps'
   → sequenceCommute qs' ps' ps qs.

(*  XXX Need to define this properly *)
Parameter conflictsWith : ∀ {pu_type : NameSet → NameSet → Type}
                                      {ppu : PartPatchUniverse pu_type pu_type}
                                      {pui : PatchUniverseInv ppu ppu}
                                      {pu : PatchUniverse pui}
                                      {ipl : InvertiblePatchlike pu}
                                      {from : NameSet},
                                ContextedPatch pu from →
                                ContextedPatch pu from →
                                Prop.

(*  XXX Need to define this properly *)
Parameter conflictsNames : ∀ {pu_type : NameSet → NameSet → Type}
                                       {ppu : PartPatchUniverse pu_type pu_type}
                                       {pui : PatchUniverseInv ppu ppu}
                                       {pu : PatchUniverse pui}
                                       {ipl : InvertiblePatchlike pu}
                                       {from : NameSet}
                                       (conflicts : list (ContextedPatch pu from)),
                                  SignedNameSet.

(*  p p^, {:p}, :q <-> q q^, {:q}, :p  *)
Inductive CatchCommute1 {pu_type : NameSet → NameSet → Type}
                           {ppu : PartPatchUniverse pu_type pu_type}
                           {pui : PatchUniverseInv ppu ppu}
                           {pu : PatchUniverse pui}
                           {ipl : InvertiblePatchlike pu}
         : ∀ {from mid1 mid2 to : NameSet},
             Catch ipl from mid1
         → Catch ipl mid1 to
         → Catch ipl from mid2
         → Catch ipl mid2 to
```

```
      → Prop
    := MkCatchCommute1 :
         ∀ {from to1 to2 : NameSet}
                {nilContains invPContains invQContains : SignedNameSet}
                (p : pu_type from to1)
                (q : pu_type from to2)
                (namesDifferent : pu_nameOf p ≠ pu_nameOf q)
                (do_not_commute : ~(commutable (invert q) p))
                (nilOK : NilOK from from nilContains)
                (invPConsOK : ConsOK to1
                                    invPContains
                                    nilContains
                                    (invert p))
                (invQConsOK : ConsOK to2
                                    invQContains
                                    nilContains
                                    (invert q)),
         CatchCommute1 (MkCatch p)
                       (Conflictor (invert p :> [])
                                   (cons (MkContextedPatch _ _ [] p) nil)
                                   (MkContextedPatch _ _ [] q))
                       (MkCatch q)
                       (Conflictor (invert q :> [])
                                   (cons (MkContextedPatch _ _ [] q) nil)
                                   (MkContextedPatch _ _ [] p)).
Notation "« p , q » <~>1 « q' , p' »"
    := (CatchCommute1 p q q' p')
    (at level 60, no associativity).

Inductive CatchCommute2 {pu_type : NameSet → NameSet → Type}
                        {ppu : PartPatchUniverse pu_type pu_type}
                        {pui : PatchUniverseInv ppu ppu}
                        {pu : PatchUniverse pui}
                        {ipl : InvertiblePatchlike pu}
       : ∀ {from mid1 mid2 to : NameSet},
          Catch ipl from mid1
       → Catch ipl mid1 to
       → Catch ipl from mid2
       → Catch ipl mid2 to
       → Prop
    := MkCatchCommute2 :
         (*  p p^ r, {r^:p} U X, y <-> r, X, y  , {y}, r^:p *)
         ∀ {from mid to : NameSet}
                {qEffectContains pEffect'Contains qEffect'Contains : SignedNameSet}
                (p : pu_type from mid)
                (qEffect : Sequence pu qEffectContains mid to)
                (qConflicts : list (ContextedPatch pu to))
                (qIdentity : ContextedPatch pu to)
                (qEffect' : Sequence pu qEffect'Contains from to)
                (qConflicts' : list (ContextedPatch pu to))
                (qIdentity' : ContextedPatch pu to)
                (pEffect' : Sequence pu pEffect'Contains to to)
                (pConflicts' : list (ContextedPatch pu to))
```

```
                    (pIdentity' : ContextedPatch pu to)
                    (namesDifferent : pu_nameOf p ≠ contextedPatch_name qIdentity)
                    (invPRsConsOK : ConsOK mid
                                        qEffectContains
                                        qEffect'Contains
                                        (invert p))
                    (nilOK : NilOK to to pEffect'Contains)
                  ,
          «qEffect» <~~>* «invert p :> qEffect'»
        → qConflicts = (MkContextedPatch _ _ (invertSequence qEffect') p) :: qConflicts'
        → qIdentity' = qIdentity
        → pEffect' = []
        → pConflicts' = qIdentity :: nil
        → pIdentity' = MkContextedPatch _ _ (invertSequence qEffect') p
        → ~(qConflicts' = nil)
        → CatchCommute2 (MkCatch p)
                        (Conflictor qEffect
                                    qConflicts
                                    qIdentity)
                        (Conflictor qEffect'
                                    qConflicts'
                                    qIdentity')
                        (Conflictor pEffect'
                                    pConflicts'
                                    pIdentity').
Notation "« p , q » <~>2 « q' , p' »"
    := (CatchCommute2 p q q' p')
    (at level 60, no associativity).
Inductive CatchCommute3 {pu_type : NameSet → NameSet → Type}
                        {ppu : PartPatchUniverse pu_type pu_type}
                        {pui : PatchUniverseInv ppu ppu}
                        {pu : PatchUniverse pui}
                        {ipl : InvertiblePatchlike pu}
      : ∀ {from mid1 mid2 to : NameSet},
        Catch ipl from mid1
      → Catch ipl mid1 to
      → Catch ipl from mid2
      → Catch ipl mid2 to
      → Prop
    := MkCatchCommute3 :
          (*  r, X, y , {y}, r^:q <-> q q^ r, {r^:q} U X, y *)
          ∀ {from mid to : NameSet}
                  {pEffectContains qEffectContains pEffect'Contains : SignedNameSet}
                  (pEffect : Sequence pu pEffectContains from to)
                  (pConflicts : list (ContextedPatch pu to))
                  (pIdentity : ContextedPatch pu to)
                  (qEffect : Sequence pu qEffectContains to to)
                  (qConflicts : list (ContextedPatch pu to))
                  (qIdentity : ContextedPatch pu to)
                  (q' : pu_type from mid)
                  (pEffect' : Sequence pu pEffect'Contains mid to)
                  (pConflicts' : list (ContextedPatch pu to))
```

```
                    (pIdentity' : ContextedPatch pu to)
                    (namesDifferent : contextedPatch_name pIdentity ≠ contextedPatch_name
qIdentity)
                    (invPRsConsOK : ConsOK mid
                                        pEffect'Contains
                                        pEffectContains
                                        (invert q'))
                    (nilOK : NilOK to to qEffectContains)
                  ,
              qEffect = []
          → qConflicts = pIdentity :: nil
          → qIdentity = MkContextedPatch _ _ (invertSequence pEffect) q'
          → «pEffect'» <~~>* «invert q' :> pEffect»
          → pConflicts' = (MkContextedPatch _ _ (invertSequence pEffect) q') :: pConflicts
          → pIdentity' = pIdentity
          → ~(pConflicts = nil)
          → CatchCommute3 (Conflictor pEffect
                                        pConflicts
                                        pIdentity)
                          (Conflictor qEffect
                                        qConflicts
                                        qIdentity)
                          (MkCatch q')
                          (Conflictor pEffect'
                                        pConflicts'
                                        pIdentity').
Notation "« p , q » <~>3 « q' , p' »"
     := (CatchCommute3 p q q' p')
     (at level 60, no associativity).
Inductive CatchCommute4 {pu_type : NameSet → NameSet → Type}
                        {ppu : PartPatchUniverse pu_type pu_type}
                        {pui : PatchUniverseInv ppu ppu}
                        {pu : PatchUniverse pui}
                        {ipl : InvertiblePatchlike pu}
         : ∀ {from mid1 mid2 to : NameSet},
           Catch ipl from mid1
         → Catch ipl mid1 to
         → Catch ipl from mid2
         → Catch ipl mid2 to
         → Prop
      := MkCatchCommute4 :
(*
            (*  r s, W, x t, {t^x} U Y, z <-> r t', s'Y, s'z s', z U t^W,
t^x *)
*)
           ∀ {o or ors orst ort : NameSet}
                    {pEffectContains qEffectContains : SignedNameSet}
                    {pEffect'Contains qEffect'Contains : SignedNameSet}
                    {commonEffectContains pOnlyEffectContains qOnlyEffectContains : Signed-
NameSet}

                    (pEffect : Sequence pu pEffectContains o ors)
```

```
                    (pConflicts : list (ContextedPatch pu ors))
                    (pIdentity : ContextedPatch pu ors)
                    (qEffect : Sequence pu qEffectContains ors orst)
                    (qConflicts : list (ContextedPatch pu orst))
                    (qOtherConflicts : list (ContextedPatch pu orst))
                    (qIdentity : ContextedPatch pu orst)
                    (qEffect' : Sequence pu qEffect'Contains o ort)
                    (qConflicts' : list (ContextedPatch pu ort))
                    (qIdentity' : ContextedPatch pu ort)
                    (pEffect' : Sequence pu pEffect'Contains ort orst)
                    (pConflicts' : list (ContextedPatch pu orst))
                    (pIdentity' : ContextedPatch pu orst)
                    (commonEffect : Sequence pu commonEffectContains o or)
                    (pOnlyEffect : Sequence pu pOnlyEffectContains or ors)
                    (qOnlyEffect : Sequence pu qOnlyEffectContains or ort)

                    (namesDifferent : contextedPatch_name pIdentity ≠ contextedPatch_name
qIdentity)

                    (appendOK_commonEffect_pOnlyEffect : appendOK commonEffect pOnly-
Effect pEffectContains)
                    (appendOK_commonEffect_qOnlyEffect : appendOK commonEffect qOnlyEf-
fect qEffect'Contains)
                  ,
          «pEffect» <~~>* «commonEffect :+> pOnlyEffect»
        → qConflicts = addSequenceToContext (invertSequence qEffect) pIdentity :: qOtherCon-
flicts
        → «qEffect'» <~~>* «commonEffect :+> qOnlyEffect»
        → qConflicts' = map (addSequenceToContext pEffect') qOtherConflicts
        → qIdentity' = addSequenceToContext pEffect' qIdentity
        → pConflicts' = qIdentity :: map (addSequenceToContext (invertSequence qEffect))
pConflicts
        → pIdentity' = addSequenceToContext (invertSequence qEffect) pIdentity
        → sequenceCommute pOnlyEffect qEffect qOnlyEffect pEffect'
        → ~(pConflicts = nil)
        → ~(qConflicts' = nil)
        → CatchCommute4 (Conflictor pEffect
                                    pConflicts
                                    pIdentity)
                        (Conflictor qEffect
                                    qConflicts
                                    qIdentity)
                        (Conflictor qEffect'
                                    qConflicts'
                                    qIdentity')
                        (Conflictor pEffect'
                                    pConflicts'
                                    pIdentity').
Notation "« p , q » <~>4 « q' , p' »"
    := (CatchCommute4 p q q' p')
    (at level 60, no associativity).
Inductive CatchCommute5 {pu_type : NameSet → NameSet → Type}
```

```
                        {ppu : PartPatchUniverse pu_type pu_type}
                        {pui : PatchUniverseInv ppu ppu}
                        {pu : PatchUniverse pui}
                        {ipl : InvertiblePatchlike pu}
        : ∀ {from mid1 mid2 to : NameSet},
          Catch ipl from mid1
        → Catch ipl mid1 to
        → Catch ipl from mid2
        → Catch ipl mid2 to
        → Prop
    := MkCatchCommute5 :
          (*  Successful commute: patch/patch *)
          ∀ {from mid1 mid2 to : NameSet}
                (p : pu_type from mid1)
                (q : pu_type mid1 to)
                (q' : pu_type from mid2)
                (p' : pu_type mid2 to),
          «p, q» <~> «q', p'» →
          CatchCommute5 (MkCatch p) (MkCatch q) (MkCatch q') (MkCatch p').
Notation "« p , q » <~>5 « q' , p' »"
    := (CatchCommute5 p q q' p')
    (at level 60, no associativity).

(*  XXX The ones below here should check there isn't already a conflict *)
Inductive CatchCommute6 {pu_type : NameSet → NameSet → Type}
                        {ppu : PartPatchUniverse pu_type pu_type}
                        {pui : PatchUniverseInv ppu ppu}
                        {pu : PatchUniverse pui}
                        {ipl : InvertiblePatchlike pu}
        : ∀ {from mid1 mid2 to : NameSet},
          Catch ipl from mid1
        → Catch ipl mid1 to
        → Catch ipl from mid2
        → Catch ipl mid2 to
        → Prop
    := MkCatchCommute6 :
          (*  Successful commute: patch/conflictor *)
          ∀ {from mid1 mid2 to : NameSet}
                {qEffectContains : SignedNameSet}
                (p : pu_type from mid1)
                (qEffect : Sequence pu qEffectContains mid1 to)
                (qConflicts : list (ContextedPatch pu to))
                (qIdentity : ContextedPatch pu to)
                (qEffect' : Sequence pu qEffectContains from mid2)
                (qConflicts' : list (ContextedPatch pu mid2))
                (qIdentity' : ContextedPatch pu mid2)
                (p' : pu_type mid2 to)
                (namesDifferent : pu_nameOf p ≠ contextedPatch_name qIdentity)
                (noConflict : ¬SignedNameSetIn (pu_nameOf p) (conflictsNames qConflicts)),
          «p, qEffect» <~> «qEffect', p'» →
          CommutePast p' qIdentity qIdentity' →
          qConflicts' = map (addToContext p') qConflicts →
```

```
            ~(qConflicts = nil) →
            CatchCommute6 (MkCatch p)
                          (Conflictor qEffect
                                      qConflicts
                                      qIdentity)
                          (Conflictor qEffect'
                                      qConflicts'
                                      qIdentity')
                          (MkCatch p').
Notation "« p , q » <~>6 « q' , p' »"
    := (CatchCommute6 p q q' p')
    (at level 60, no associativity).

Inductive CatchCommute7 {pu_type : NameSet → NameSet → Type}
                        {ppu : PartPatchUniverse pu_type pu_type}
                        {pui : PatchUniverseInv ppu ppu}
                        {pu : PatchUniverse pui}
                        {ipl : InvertiblePatchlike pu}
        : ∀ {from mid1 mid2 to : NameSet},
          Catch ipl from mid1
        → Catch ipl mid1 to
        → Catch ipl from mid2
        → Catch ipl mid2 to
        → Prop
      := MkCatchCommute7 :
            (*  Successful commute: conflictor/patch *)
            ∀ {from mid1 mid2 to : NameSet}
                  {pEffectContains : SignedNameSet}
                  (pEffect : Sequence pu pEffectContains from mid1)
                  (pConflicts : list (ContextedPatch pu mid1))
                  (pIdentity : ContextedPatch pu mid1)
                  (q : pu_type mid1 to)
                  (q' : pu_type from mid2)
                  (pEffect' : Sequence pu pEffectContains mid2 to)
                  (pConflicts' : list (ContextedPatch pu to))
                  (pIdentity' : ContextedPatch pu to)
                  (namesDifferent : contextedPatch_name pIdentity ≠ pu_nameOf q),
            «pEffect, q» <~> «q', pEffect'» →
            CommutePast (invert q) pIdentity pIdentity' →
            pConflicts' = map (addToContext (invert q)) pConflicts →
            ~(pConflicts = nil) →
            CatchCommute7 (Conflictor pEffect
                                      pConflicts
                                      pIdentity)
                          (MkCatch q)
                          (MkCatch q')
                          (Conflictor pEffect'
                                      pConflicts'
                                      pIdentity').
Notation "« p , q » <~>7 « q' , p' »"
    := (CatchCommute7 p q q' p')
    (at level 60, no associativity).
```

```
Inductive CatchCommute8 {pu_type : NameSet → NameSet → Type}
                        {ppu : PartPatchUniverse pu_type pu_type}
                        {pui : PatchUniverseInv ppu ppu}
                        {pu : PatchUniverse pui}
                        {ipl : InvertiblePatchlike pu}
        : ∀ {from mid1 mid2 to : NameSet},
          Catch ipl from mid1
        → Catch ipl mid1 to
        → Catch ipl from mid2
        → Catch ipl mid2 to
        → Prop
     := MkCatchCommute8 :
          (*  Successful commute: conflictor/conflictor *)
          ∀ {from common mid1 mid2 to : NameSet}
                  {commonEffectContains : SignedNameSet}
                  {pEffectContains pOnlyEffectContains : SignedNameSet}
                  {qEffectContains qOnlyEffectContains : SignedNameSet}
                  (pEffect : Sequence pu pEffectContains from mid1)
                  (pConflicts : list (ContextedPatch pu mid1))
                  (pIdentity : ContextedPatch pu mid1)
                  (qEffect : Sequence pu qEffectContains mid1 to)
                  (qConflicts : list (ContextedPatch pu to))
                  (qIdentity : ContextedPatch pu to)
                  (qEffect' : Sequence pu qEffectContains from mid2)
                  (qConflicts' : list (ContextedPatch pu mid2))
                  (qIdentity' : ContextedPatch pu mid2)
                  (pEffect' : Sequence pu pEffectContains mid2 to)
                  (pConflicts' : list (ContextedPatch pu to))
                  (pIdentity' : ContextedPatch pu to)
                  (commonEffect : Sequence pu commonEffectContains from common)
                  (pOnlyEffect : Sequence pu pOnlyEffectContains common mid1)
                  (qOnlyEffect : Sequence pu qOnlyEffectContains common mid2)
                  (appendOK_commonEffect_pOnlyEffect : appendOK commonEffect pOnly-
Effect pEffectContains)
                  (appendOK_commonEffect_qOnlyEffect : appendOK commonEffect qOnlyEf-
fect qEffectContains)
                  (namesDifferent : contextedPatch_name pIdentity ≠ contextedPatch_name
qIdentity),

          (*  Effects *)
(*
                        pEffect                                 qEffect
    <from> commonEffect <common> pOnlyEffect <mid1>       <mid1>qEffect<to>

                        qEffect'                                pEffect'
    <from> commonEffect <common> qOnlyEffect <mid2>       <mid2>pEffect'<to>
*)
          sequenceCommute pOnlyEffect qEffect qOnlyEffect pEffect' →
          «pEffect» <~~>* «commonEffect :+> pOnlyEffect» →
          «qEffect'» <~~>* «commonEffect :+> qOnlyEffect» →
          SignedNameSetEqual
              SignedNameSetMod.empty
```

```
                (SignedNameSetMod.inter
                     pOnlyEffectContains
                     qOnlyEffectContains) →

          (*  Identities *)
          CommuteManyPast (invertSequence qEffect) pIdentity pIdentity' →
          CommuteManyPast pEffect' qIdentity qIdentity' →

          (*  Conflicts *)
          pConflicts' = map (addSequenceToContext (invertSequence qEffect)) pConflicts →
          qConflicts' = map (addSequenceToContext pEffect') qConflicts →

          (*  The patches themselves can't conflict *)
          ~(conflictsWith pIdentity (addSequenceToContext qEffect qIdentity)) →

          ~(pConflicts = nil) →
          ~(qConflicts = nil) →

          CatchCommute8 (Conflictor pEffect
                                    pConflicts
                                    pIdentity)
                        (Conflictor qEffect
                                    qConflicts
                                    qIdentity)
                        (Conflictor qEffect'
                                    qConflicts'
                                    qIdentity')
                        (Conflictor pEffect'
                                    pConflicts'
                                    pIdentity').
Notation "« p , q » <~>8 « q' , p' »"
    := (CatchCommute8 p q q' p')
    (at level 60, no associativity).
Inductive CatchCommute {pu_type : NameSet → NameSet → Type}
                       {ppu : PartPatchUniverse pu_type pu_type}
                       {pui : PatchUniverseInv ppu ppu}
                       {pu : PatchUniverse pui}
                       {ipl : InvertiblePatchlike pu}
                       {from mid1 mid2 to : NameSet}
                       (p : Catch ipl from mid1)
                       (q : Catch ipl mid1 to)
                       (q' : Catch ipl from mid2)
                       (p' : Catch ipl mid2 to)
                     : Prop
      (*  p p^, {:p}, :q <-> q q^, {:q}, :p *)
    := isCatchCommute1 : ∀ (catchCommuteDetails : CatchCommute1 p q q' p'),
                         CatchCommute p q q' p'
          (*  p p^ r, {r^:p} U X, y <-> r, X, y , {y}, r^:p *)
     | isCatchCommute2 : ∀ (catchCommuteDetails : CatchCommute2 p q q' p'),
                         CatchCommute p q q' p'
          (*  r, X, y , {y}, r^:q <-> q q^ r, {r^:q} U X, y *)
     | isCatchCommute3 : ∀ (catchCommuteDetails : CatchCommute3 p q q' p'),
```

```
                        CatchCommute p q q' p'
          (*  r s, W, x t, {t^x} U Y, z <-> r t', s'Y, s'z s', z U t^W,
t^x *)
    | isCatchCommute4 : ∀ (catchCommuteDetails : CatchCommute4 p q q' p'),
                        CatchCommute p q q' p'
          (*  Successful commute: patch/patch *)
    | isCatchCommute5 : ∀ (catchCommuteDetails : CatchCommute5 p q q' p'),
                        CatchCommute p q q' p'
          (*  Successful commute: patch/conflictor *)
    | isCatchCommute6 : ∀ (catchCommuteDetails : CatchCommute6 p q q' p'),
                        CatchCommute p q q' p'
          (*  Successful commute: conflictor/patch *)
    | isCatchCommute7 : ∀ (catchCommuteDetails : CatchCommute7 p q q' p'),
                        CatchCommute p q q' p'
          (*  Successful commute: conflictor/conflictor *)
    | isCatchCommute8 : ∀ (catchCommuteDetails : CatchCommute8 p q q' p'),
                        CatchCommute p q q' p'.
Notation "« p , q » <~>c « q' , p' »"
    := (CatchCommute p q q' p')
    (at level 60, no associativity).

(*  XXX Move this lemma to contexted_patches: *)
Lemma nameOfAddSequenceToContext :
      ∀ {pu_type : NameSet → NameSet → Type}
            {ppu : PartPatchUniverse pu_type pu_type}
            {pui : PatchUniverseInv ppu ppu}
            {pu : PatchUniverse pui}
            {ipl : InvertiblePatchlike pu}
            {from to : NameSet}
            {psContains : SignedNameSet}
            {ps : Sequence pu psContains from to}
            {cp : ContextedPatch pu to},
      contextedPatch_name (addSequenceToContext ps cp) = contextedPatch_name cp.
Proof with auto.
intros.
induction ps.
    admit.
admit.
Qed.

(*  XXX *)
Lemma addSequenceToContextInverse1 :
      ∀ {pu_type : NameSet → NameSet → Type}
            {ppu : PartPatchUniverse pu_type pu_type}
            {pui : PatchUniverseInv ppu ppu}
            {pu : PatchUniverse pui}
            {ipl : InvertiblePatchlike pu}
            {from to : NameSet}
            {psContains : SignedNameSet}
            {ps : Sequence pu psContains from to}
            {cp : ContextedPatch pu from},
      addSequenceToContext ps
          (addSequenceToContext (invertSequence ps) cp) = cp.
```

```
Proof with auto.
intros.
induction ps.
    admit.
admit.
Qed.

Lemma addSequenceToContextInverse2 :
      ∀ {pu_type : NameSet → NameSet → Type}
              {ppu : PartPatchUniverse pu_type pu_type}
              {pui : PatchUniverseInv ppu ppu}
              {pu : PatchUniverse pui}
              {ipl : InvertiblePatchlike pu}
              {from to : NameSet}
              {psContains : SignedNameSet}
              {ps : Sequence pu psContains from to}
              {cp : ContextedPatch pu to},
      addSequenceToContext (invertSequence ps)
          (addSequenceToContext ps cp) = cp.
Proof with auto.
intros.
induction ps.
    admit.
admit.
Qed.

Lemma addSequenceToContextIdentity1 :
      ∀ {pu_type : NameSet → NameSet → Type}
              {ppu : PartPatchUniverse pu_type pu_type}
              {pui : PatchUniverseInv ppu ppu}
              {pu : PatchUniverse pui}
              {ipl : InvertiblePatchlike pu}
              {from to : NameSet}
              {psContains : SignedNameSet}
              {ps : Sequence pu psContains from to},
      ∀ (cp : ContextedPatch pu from),
      (fun x ⇒
            addSequenceToContext ps
                (addSequenceToContext (invertSequence ps) x)) cp
    = (fun x ⇒ x) cp.
Proof with auto.
intros.
simpl.
apply addSequenceToContextInverse1...
Qed.

Lemma addSequenceToContextIdentity2 :
      ∀ {pu_type : NameSet → NameSet → Type}
              {ppu : PartPatchUniverse pu_type pu_type}
              {pui : PatchUniverseInv ppu ppu}
              {pu : PatchUniverse pui}
              {ipl : InvertiblePatchlike pu}
              {from to : NameSet}
              {psContains : SignedNameSet}
```

```
                {ps : Sequence pu psContains from to},
        ∀ (cp : ContextedPatch pu to),
        (fun x ⇒
              addSequenceToContext (invertSequence ps)
                 (addSequenceToContext ps x)) cp
     = (fun x ⇒ x) cp.
Proof with auto.
intros.
simpl.
apply addSequenceToContextInverse2...
Qed.

(*  XXX Move this lemma to contexted_patches: *)
Lemma nameOfCommutePast :
       ∀ {pu_type : NameSet → NameSet → Type}
                {ppu : PartPatchUniverse pu_type pu_type}
                {pui : PatchUniverseInv ppu ppu}
                {pu : PatchUniverse pui}
                {ipl : InvertiblePatchlike pu}
                {from to : NameSet}
                {p : pu_type from to}
                {cp : ContextedPatch pu to}
                {cp' : ContextedPatch pu from},
        CommutePast p cp cp'
    → contextedPatch_name cp' = contextedPatch_name cp.
Proof with auto.
intros.
induction H.
    simpl.
    destruct (commuteNames H) as [? [? ?]]...
simpl in ×...
Qed.

(*  XXX Move this lemma to contexted_patches: *)
Lemma nameOfCommuteManyPast :
       ∀ {pu_type : NameSet → NameSet → Type}
                {ppu : PartPatchUniverse pu_type pu_type}
                {pui : PatchUniverseInv ppu ppu}
                {pu : PatchUniverse pui}
                {ipl : InvertiblePatchlike pu}
                {from to : NameSet}
                {psContains : SignedNameSet}
                {ps : Sequence pu psContains from to}
                {cp : ContextedPatch pu to}
                {cp' : ContextedPatch pu from},
        CommuteManyPast ps cp cp'
    → contextedPatch_name cp' = contextedPatch_name cp.
Proof with auto.
intros.
induction H...
rewrite ← IHCommuteManyPast.
apply (nameOfCommutePast CommutePPast).
Qed.
```

```
(*  XXX Move this lemma?: *)
Lemma nameOfCommuteOneMany :
      ∀ {pu_type : NameSet → NameSet → Type}
              {ppu : PartPatchUniverse pu_type pu_type}
              {pui : PatchUniverseInv ppu ppu}
              {pu : PatchUniverse pui}
              {from mid1 mid2 to : NameSet}
              {qsContains : SignedNameSet}
              {p : pu_type from mid1}
              {qs : Sequence pu qsContains mid1 to}
              {qs' : Sequence pu qsContains from mid2}
              {p' : pu_type mid2 to},
      «p, qs» <~> «qs', p'»
   → pu_nameOf p = pu_nameOf p'.
Proof with auto.
Admitted.
(*
intros.
induction H...
rewrite <- IHcommuteOneMany.
destruct (commuteNames pq_commute_q'p')...
Qed.
*)

(*  XXX Move this lemma?: *)
Lemma nameOfCommuteManyOne :
      ∀ {pu_type : NameSet → NameSet → Type}
              {ppu : PartPatchUniverse pu_type pu_type}
              {pui : PatchUniverseInv ppu ppu}
              {pu : PatchUniverse pui}
              {ipl : InvertiblePatchlike pu}
              {from mid1 mid2 to : NameSet}
              {psContains : SignedNameSet}
              {ps : Sequence pu psContains from mid2}
              {q : pu_type mid2 to}
              {q' : pu_type from mid1}
              {ps' : Sequence pu psContains mid1 to},
      «ps, q» <~> «q', ps'»
   → pu_nameOf q = pu_nameOf q'.
Proof with auto.
Admitted.
(*
intros.
induction H...
rewrite IHcommuteManyOne.
destruct (commuteNames _ pr'_commute_r"p') as ?  [?  ?]...
Qed.
*)

Lemma CatchCommuteNames :
      ∀ {pu_type : NameSet → NameSet → Type}
              {ppu : PartPatchUniverse pu_type pu_type}
              {pui : PatchUniverseInv ppu ppu}
```

```
                {pu : PatchUniverse pui}
                {ipl : InvertiblePatchlike pu}
                {from mid1 mid2 to : NameSet}
                {p : Catch ipl from mid1} {q : Catch ipl mid1 to}
                {q' : Catch ipl from mid2} {p' : Catch ipl mid2 to}
                (c : « p , q » <~>c « q' , p' »),
        (catch_name p = catch_name p') ∧
        (catch_name q = catch_name q') ∧
        (catch_name p ≠ catch_name q).
Proof with auto.
intros.
dependent destruction c; dependent destruction catchCommuteDetails; simpl.
                                split...
                            split...
                                dependent destruction pIdentity'.
                                simpl...
                            split...
                            dependent destruction qIdentity'.
                            simpl...
                        split...
                            dependent destruction pIdentity'...
                        split...
                        dependent destruction qIdentity...
                    split...
                        dependent destruction pIdentity.
                        rewrite nameOfAddSequenceToContext...
                    split...
                    dependent destruction qIdentity.
                    rewrite nameOfAddSequenceToContext...
                apply commuteNames...
            split...
                admit.
                (*
                dependent destruction H...
                destruct (commuteNames _ pq_commute_q'p').
                rewrite H1.
                apply (nameOfCommuteOneMany H).
                *)
            split...
            apply nameOfCommutePast in H0...
        split...
            rewrite (nameOfCommutePast H0)...
        split...
        apply nameOfCommuteManyOne in H...
split...
    apply nameOfCommuteManyPast in H3...
split...
apply nameOfCommuteManyPast in H4...
Qed.
Definition CatchCommutable {pu_type : NameSet → NameSet → Type}
                           {ppu : PartPatchUniverse pu_type pu_type}
                           {pui : PatchUniverseInv ppu ppu}
```

```
                                {pu : PatchUniverse pui}
                                {ipl : InvertiblePatchlike pu}
                                {from mid1 to : NameSet}
                                (p : Catch ipl from mid1)
                                (q : Catch ipl mid1 to) : Prop
 := ∃ mid2 : NameSet,
    ∃ q' : Catch ipl from mid2,
    ∃ p' : Catch ipl mid2 to,
    «p, q» <~>c «q', p'».
Notation "p <~?~>c q" := (CatchCommutable p q)
    (at level 60, no associativity).

Lemma CatchCommutable_dec :
      ∀ {pu_type : NameSet → NameSet → Type}
              {ppu : PartPatchUniverse pu_type pu_type}
              {pui : PatchUniverseInv ppu ppu}
              {pu : PatchUniverse pui}
              {ipl : InvertiblePatchlike pu}
              {from mid to : NameSet}
              (p : Catch ipl from mid)
              (q : Catch ipl mid to),
      {p <~?~>c q} + {~(p <~?~>c q)}.
Proof with auto.
intros.
destruct p; destruct q.
            (*  Two patches case *)
            destruct (commutable_dec p p0).
                left.
                destruct c as [mid' [q' [p' c']]].
                ∃ mid'.
                ∃ (MkCatch q').
                ∃ (MkCatch p').
                apply isCatchCommute5.
                apply MkCatchCommute5...
            right.
            intro c.
            destruct c as [mid' [q' [p' c']]].
            destruct c'; dependent destruction catchCommuteDetails.
            elim n.
            ∃ mid2.
            ∃ q'.
            ∃ p'...
        (*  Patch/Conflictor case *)
        admit.
    (*  Conflictor/Patch case *)
    admit.
(*  Conflictor/Conflictor case *)
admit.
Qed.

Lemma CatchCommuteSelfInverse :
      ∀ {pu_type : NameSet → NameSet → Type}
              {ppu : PartPatchUniverse pu_type pu_type}
```

```
                {pui : PatchUniverseInv ppu ppu}
                {pu : PatchUniverse pui}
                {ipl : InvertiblePatchlike pu}
                {from mid1 mid2 to : NameSet}
                {p : Catch ipl from mid1}
                {q : Catch ipl mid1 to}
                {q' : Catch ipl from mid2}
                {p' : Catch ipl mid2 to}
                (c : «p, q» <~>c «q', p'»),
        («q', p'» <~>c «p, q»).
Proof with auto.
intros.
destruct c; destruct catchCommuteDetails.
                                (*  MkCatchCommute1 case *)
                                apply isCatchCommute1.
                                apply MkCatchCommute1.
                                    apply sym_not_eq...
                                intro.
                                destruct H as [mid [q' [p' Hcommute]]].
                                apply commuteInverses in Hcommute.
                                rewrite invertInverse in Hcommute.
                                elim do_not_commute.
                                ∃ mid.
                                ∃ (invert p').
                                ∃ (invert q')...
                            (*  MkCatchCommute2 case *)
                            subst.
                            apply isCatchCommute3.
                            eapply MkCatchCommute3...
                        (*  MkCatchCommute3 case *)
                        subst.
                        apply isCatchCommute2.
                        eapply MkCatchCommute2...
                    (*  MkCatchCommute4 case *)
                    apply isCatchCommute4.
                    refine (MkCatchCommute4
                                qEffect' qConflicts' qIdentity'
                                pEffect' pConflicts' (map (addSequenceToContext (invertSequence qEffect)) pConflicts) pIdentity'
                                pEffect pConflicts pIdentity
                                qEffect qConflicts qIdentity
                                commonEffect _ _ _ _ _ _ _ _ _ _ _ _ _ _ _)...
                                                subst.
                                                rewrite nameOfAddSequenceToContext.
                                                rewrite nameOfAddSequenceToContext...
                                            apply H1.
                                        subst.
                                        rewrite addSequenceToContextInverse2...
                                    apply H.
                                rewrite map_map.
                                rewrite (map_ext _ _ addSequenceToContextIdentity1).
                                rewrite map_id...
```

```
                        subst.
                        rewrite addSequenceToContextInverse1...
                    subst.
                    rewrite map_map.
                    rewrite (map_ext _ _ addSequenceToContextIdentity2).
                    rewrite map_id...
                subst.
                rewrite addSequenceToContextInverse2...
            apply sequenceCommuteInverse...
        (*  MkCatchCommute5 case *)
        apply isCatchCommute5.
        apply MkCatchCommute5.
        apply commuteSelfInverse in H...
    (*  MkCatchCommute6 case *)
    apply isCatchCommute7.
    apply MkCatchCommute7.
                    rewrite ← (nameOfCommuteOneMany H).
                    rewrite (nameOfCommutePast H0).
                    apply sym_not_eq...
                admit.
                (*
                 apply commute_OneMany_ManyOne...
                 *)
            admit.
        admit.
    subst.
    apply map_neq_nil...
(*  MkCatchCommute7 case *)
apply isCatchCommute6.
apply MkCatchCommute6.
                    rewrite ← (nameOfCommuteManyOne H).
                    rewrite (nameOfCommutePast H0).
                    apply sym_not_eq...
                admit.
            admit.
            (*
             apply commute_ManyOne_OneMany...
             *)
        admit.
    admit.
subst.
apply map_neq_nil...
(*  MkCatchCommute8 case *)
apply isCatchCommute8.
eapply MkCatchCommute8...
                                rewrite (nameOfCommuteManyPast H3).
                                rewrite (nameOfCommuteManyPast H4)...
                            apply sequenceCommuteInverse...
                        clear - H2.
                        signedNameSetDec.
                    admit.
                admit.
```

```
                admit.
            admit.
        admit.
    subst.
    apply map_neq_nil...
subst.
apply map_neq_nil...
Qed.

Lemma CatchCommuteUnique1 :
      ∀ {pu_type : NameSet → NameSet → Type}
            {ppu : PartPatchUniverse pu_type pu_type}
            {pui : PatchUniverseInv ppu ppu}
            {pu : PatchUniverse pui}
            {ipl : InvertiblePatchlike pu}
            {from mid mid' mid" to : NameSet}
            {p : Catch ipl from mid} {q : Catch ipl mid to}
            {q' : Catch ipl from mid'} {p' : Catch ipl mid' to}
            {q" : Catch ipl from mid"} {p" : Catch ipl mid" to}
            (commute1 : «p, q» <~>c «q', p'»)
            (commute2 : «p, q» <~>c «q", p"»),
      (mid' = mid").
Proof with auto.
intros.
dependent destruction commute1;
dependent destruction catchCommuteDetails;
dependent destruction commute2;
dependent destruction catchCommuteDetails.
(*  Rule 1 / Rule 1 *)
auto.
(*  Rule 1 / Rule 2 *)
congruence.
(*  Rule 1 / Rule 6 *)
(*
noConflict : ~
             SignedNameSetIn (pu_nameOf ipl p)
               (conflictsNames (MkContextedPatch ipl from  p :: nil))
*)
admit. (*  Contradiction in noConflict *)
(*  Rule 2 / Rule 1 *)
congruence.
(*  Rule 2 / Rule 2 *)
auto.
(*  Rule 2 / Rule 6 *)
subst.
(*
noConflict : ~
             SignedNameSetIn (pu_nameOf ipl p)
               (conflictsNames
                  (MkContextedPatch ipl to (invertSequence qEffect') p
                   :: qConflicts'))
*)
admit. (*  Contradiction in noConflict *)
```

```
(*  Rule 3 / Rule 3 *)
auto.
(*  Rule 3 / Rule 4 *)
apply map_neq_nil2 in H15.
congruence.
(*  Rule 3 / Rule 8 *)
subst.
admit.
(*  Rule 4 / Rule 3 *)
apply map_neq_nil2 in H8.
congruence.
(*  Rule 4 / Rule 4 *)
subst.
admit.
(*  Rule 4 / Rule 8 *)
subst.
admit.
(*  Rule 5 / Rule 5 *)
apply (commuteUniqueTypes H H0).
(*  Rule 6 / Rule 1 *)
(*
noConflict : ~
             SignedNameSetIn (pu_nameOf ipl p)
               (conflictsNames (MkContextedPatch ipl from  p :: nil))
*)
admit. (*  Contradiction in noConflict *)
(*  Rule 6 / Rule 2 *)
(*
noConflict : ~
             SignedNameSetIn (pu_nameOf ipl p)
               (conflictsNames
                  (MkContextedPatch ipl to (invertSequence qEffect'0) p
                   :: qConflicts'0))
*)
admit. (*  Contradiction in noConflict *)
(*  Rule 6 / Rule 6 *)
admit.
(*  Rule 7 / Rule 7 *)
admit.
(*  Rule 8 / Rule 3 *)
admit.
(*  Rule 8 / Rule 4 *)
admit.
(*  Rule 8 / Rule 8 *)
admit.
Qed.
Lemma CatchCommuteUnique2 :
      ∀ {pu_type : NameSet → NameSet → Type}
            {ppu : PartPatchUniverse pu_type pu_type}
            {pui : PatchUniverseInv ppu ppu}
            {pu : PatchUniverse pui}
            {ipl : InvertiblePatchlike pu}
```

```
                {from mid mid' to : NameSet}
                {p : Catch ipl from mid} {q : Catch ipl mid to}
                {q' : Catch ipl from mid'} {p' : Catch ipl mid' to}
                {q" : Catch ipl from mid'} {p" : Catch ipl mid' to}
                (commute1 : «p, q» <~>c «q', p'»)
                (commute2 : «p, q» <~>c «q", p"»),
        (p' = p") ∧ (q' = q").
Proof with auto.
intros.
dependent destruction commute1;
dependent destruction catchCommuteDetails;
dependent destruction commute2;
dependent destruction catchCommuteDetails.
(*  Rule 1 / Rule 1 *)
split...
assert (HX1 : nilOK = nilOK0).
    apply proof_irrelevance.
rewrite HX1.
assert (HX2 : invQContains = invQContains0).
    clear - invQConsOK invQConsOK0.
    destruct invQConsOK.
    destruct invQConsOK0.
    apply SignedNameSetEquality.
    remember (pu_nameOf (q^)) as HX. (*  Workaround for coq 2464 *)
    signedNameSetDec.
subst.
assert (HX2 : invQConsOK = invQConsOK0).
    apply proof_irrelevance.
rewrite HX2...
(*  Rule 1 / Rule 2 *)
congruence.
(*  Rule 1 / Rule 6 *)
admit.
(*  Others... *)
congruence.
admit.
admit.
admit.
admit.
admit.
admit.
admit.
admit.
admit.
admit.
admit.
admit.
admit.
admit.
admit.
admit.
admit.
admit.
admit.
(*
```

```
destruct (commuteUnique _ H H0).
subst.
split...
admit.
*)
Qed.
```

End *catches_definition*.

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12 Catches and Repos

```
Module Export catches.

Require Import Equality.
Require Import names.
Require Import patch_universes.
Require Import invertible_patchlike.
Require Import contexted_patches.
Require Import catches_definition.
Require Import catches_commute_consistent.

Lemma CatchCommuteConsistent2 :
      ∀ {pu_type : NameSet → NameSet → Type}
              {ppu : PartPatchUniverse pu_type pu_type}
              {pui : PatchUniverseInv ppu ppu}
              {pu : PatchUniverse pui}
              (ipl : InvertiblePatchlike pu)
              {o op opq opqr or oq oqr : NameSet}
              {q3 : Catch ipl o oq}
              {r3 : Catch ipl oq oqr}
              {p3 : Catch ipl oqr opqr}
              {q4 : Catch ipl or oqr}
              {r4 : Catch ipl o or}
              {p1 : Catch ipl o op}
              {q1 : Catch ipl op opq}
              {r1 : Catch ipl opq opqr}
              {p5 : Catch ipl oq opq},
      «q3, r3» <˜>c «r4, q4»
   → «r3, p3» <˜>c «p5, r1»
   → «q3, p5» <˜>c «p1, q1»
   → (∃ opr : NameSet,
       (∃ r2 : Catch ipl op opr,
        (∃ q2 : Catch ipl opr opqr,
         (∃ p6 : Catch ipl or opr,
          «q1, r1» <˜>c «r2, q2» ∧
          «q4, p3» <˜>c «p6, q2» ∧
```

```
          «r4, p6» <~>c «p1, r2»))))).
Proof with auto.
intros.
(*
dependent destruction H; dependent destruction H0; dependent destruction H1...

XXX.



    destruct (commuteNames _ H) as [HX1 [HX2 HX3]].
    admit.
(*     congruence. *)
destruct (commuteConsistent1 _ H H0 H1) as [r4 [q4 [p6 [H2 [H3 H4]]]]].
exists or.
exists (MkCatch r4).
exists (MkCatch q4).
exists (MkCatch p6).
split.
    apply MkCatchCommute...
split.
    apply MkCatchCommute...
apply MkCatchCommute...

(*  XXX *)
*)
admit.
Qed.

Instance CatchPartPatchUniverse
             {pu_type : NameSet → NameSet → Type}
             {ppu : PartPatchUniverse pu_type pu_type}
             {pui : PatchUniverseInv ppu ppu}
             {pu : PatchUniverse pui}
             (ipl : InvertiblePatchlike pu)
           : PartPatchUniverse (Catch ipl) (Catch ipl)
    := mkPartPatchUniverse
           (Catch ipl)
           (Catch ipl)
           (@CatchCommute _ _ _ _ _)
           (@CatchCommutable_dec _ _ _ _ _)
           (@CatchCommuteUnique1 _ _ _ _ _)
           (@CatchCommuteUnique2 _ _ _ _ _).

Instance CatchPatchUniverseInv
             {pu_type : NameSet → NameSet → Type}
             {ppu : PartPatchUniverse pu_type pu_type}
             {pui : PatchUniverseInv ppu ppu}
             {pu : PatchUniverse pui}
             (ipl : InvertiblePatchlike pu)
           : PatchUniverseInv (CatchPartPatchUniverse ipl) (CatchPartPatchUniverse ipl)
    := mkPatchUniverseInv
           (Catch ipl)
```

```
            (Catch ipl)
            (CatchPartPatchUniverse ipl)
            (CatchPartPatchUniverse ipl)
            (@CatchCommuteSelfInverse _ _ _ _ _).
Instance CatchPatchUniverse
              {pu_type : NameSet → NameSet → Type}
              {ppu : PartPatchUniverse pu_type pu_type}
              {pui : PatchUniverseInv ppu ppu}
              {pu : PatchUniverse pui}
              (ipl : InvertiblePatchlike pu)
            : PatchUniverse (CatchPatchUniverseInv ipl)
    := mkPatchUniverse
            (Catch ipl)
            (CatchPartPatchUniverse ipl)
            (CatchPatchUniverseInv ipl)
            (@catch_name _ _ _ _ _)
            (@CatchCommuteConsistent1 _ _ _ _ _)
            (@CatchCommuteConsistent2 _ _ _ _ _)
            (@CatchCommuteNames _ _ _ _ _).
End catches.
```

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13 Catch Patch Universe

```
Module Export catch_patch_universe.
Require Import hunks.
Require Import catches.
Require Import catches_definition.
Require Import patch_universes.
Require Import names.
Definition HunkCatchType := Catch HunkInvertiblePatchlike.
Definition HunkCatchPartPatchUniverse := CatchPartPatchUniverse HunkInvertiblePatchlike.
Definition HunkCatchPatchUniverseInv := CatchPatchUniverseInv HunkInvertiblePatchlike.
Definition HunkCatchPatchUniverse := CatchPatchUniverse HunkInvertiblePatchlike.
Lemma hunkCatchCommuteInSequenceConsistent :
    ∀ {o oq oqr oqrs ou our ours : NameSet}
            {qsContains : SignedNameSet}
            {qs : Sequence HunkCatchPatchUniverse qsContains o oq}
            {r : HunkCatchType oq oqr}
            {s : HunkCatchType oqr oqrs}
            (*  ts omitted for now *)
            {usContains vsContains : SignedNameSet}
            {us : Sequence HunkCatchPatchUniverse usContains o ou}
            {r' : HunkCatchType ou our}
```

```
            {s' : HunkCatchType our ours}
            {vs : Sequence HunkCatchPatchUniverse vsContains ours oqrs}

            {NilContains s_NilContains r_s_NilContains : SignedNameSet}
            {s'_vsContains r'_s'_vsContains : SignedNameSet}
            {contains : SignedNameSet}

            (nilOK : NilOK oqrs oqrs NilContains)
            (s_NilConsOK : ConsOK oqr s_NilContains NilContains s)
            (r_s_NilConsOK : ConsOK oq r_s_NilContains s_NilContains r)
            (qs_r_s_NilAppendOK : appendOK qs (r :> s :> []) contains)

            (s'_vsConsOK : ConsOK our s'_vsContains vsContains s')
            (r'_s'_vsConsOK : ConsOK ou r'_s'_vsContains s'_vsContains r')
            (us_r'_s'_vsAppendOK : appendOK us (r' :> (s' :> vs)) contains),
    («qs :+> r :> s :> []» <~~>* «us :+> r' :> s' :> vs»)
 → (catch_name r = catch_name r')
 → (catch_name s = catch_name s')
 → (r <~?~> s)
 → (r' <~?~> s').
Proof.
apply (@commuteInSequenceConsistent HunkCatchType HunkCatchPartPatchUniverse
HunkCatchPatchUniverseInv HunkCatchPatchUniverse).
Qed.
```

$[p]\,([a]\,[b] + [c]\,[d]) = [p]\,[a]\,[b]\left[b^{-1}a^{-1}, \{: a, a : b\}, : c\right][\epsilon, \{: a, a : b\}, c : d]$

You can trace a sequence $pabb^{-1}a^{-1}ab$

XXXXXXXXXXXXXXXXXXXXXX

XXX Want to show that catches inhabit a patch universe

XXX This all need to be updated properly for catches

XXX At some point we need to say that equality on catches is only up to commutation of their internals

Lemma 13.1 (catch-commute-unique)
$\forall p \in \mathbf{P}, q \in \mathbf{P}, j \in (\mathbf{P} \times \mathbf{P}) \cup \{\text{fail}\}, k \in (\mathbf{P} \times \mathbf{P}) \cup \{\text{fail}\} \cdot$
$(\langle p, q\rangle \hookrightarrow j) \wedge (\langle p, q\rangle \hookrightarrow k) \Rightarrow j = k$

Explanation
This is Lemma 7.1 restated for catches.

Proof
Catch commute is a load of patch commutes. Each of those patch commutes has a unique result by Lemma 7.1. ∎

XXX This needs to be strengthened to only talk about catches adjacent in a repo:

Lemma 13.2 (catch-commute-self-inverse)
$\forall p \in \mathbf{P}, q \in \mathbf{P}, p' \in \mathbf{P}, q' \in \mathbf{P} \cdot$
$(\langle p, q\rangle \hookrightarrow \langle q', p'\rangle) \Leftrightarrow (\langle q', p'\rangle \hookrightarrow \langle p, q\rangle)$

Explanation
This is Lemma 7.2 restated for catches.

Proof
XXX Need names/numbers for the rules.

If two catches commutes by the patch/conflicting-patch rule, then they commute back by the same rule.

If two catches commutes by the conflictor/conflicting-patch rule, then they commute back by the patch/conflicting-conflictor rule, and vice-versa.

For the conflictor/conflicting-conflictor rule, in the forward direction we have
$\left\langle \left[\overline{rs}, W, x\right], \left[\overline{t}, \left\{\overline{t}^{-1}x\right\} \cup Y, z\right]\right\rangle \leftrightarrow \left\langle \left[\overline{rt'}, \overline{s'}Y, \overline{s'}z\right], \left[\overline{s'}, \{z\} \cup \overline{t}^{-1}W, \overline{t}^{-1}x\right]\right\rangle$
Adding an identity (XXX need a lemma that this is an identity) and some parentheses to the right hand side gives us
$\left\langle \left[\overline{rt'}, (\overline{s'}Y), (\overline{s'}z)\right], \left[\overline{s'}, \left\{\overline{s'}^{-1}(\overline{s'}z)\right\} \cup (\overline{t}^{-1}W), (\overline{t}^{-1}x)\right]\right\rangle$
and by the conflictor/conflicting-conflictor rule we get
$\left\langle \left[\overline{rt'}, (\overline{s'}Y), (\overline{s'}z)\right], \left[\overline{s'}, \left\{\overline{s'}^{-1}(\overline{s'}z)\right\} \cup (\overline{t}^{-1}W), (\overline{t}^{-1}x)\right]\right\rangle \leftrightarrow$
$\left\langle \left[\overline{rs}, \overline{t}(\overline{t}^{-1}W), \overline{t}(\overline{t}^{-1}x)\right], \left[\overline{t}, \left\{(\overline{t}^{-1}x)\right\} \cup \overline{s'}^{-1}(\overline{s'}Y), \overline{s'}^{-1}(\overline{s'}z)\right]\right\rangle$
if $\left\langle \overline{t'}, \overline{s'}\right\rangle \leftrightarrow \left\langle \overline{s}, \overline{t}\right\rangle$
$N\left(\overline{r}^{-1}\right) \subseteq N\left(\overline{t}^{-1}W\right))$
$N\left(\overline{t'}^{-1}\right) \cap N\left(\overline{t}^{-1}W\right)) = \emptyset$
XXX Show side conditions OK
Finally, simplifying the result (XXX need a lemma for that too) gives us
$\left\langle \left[\overline{rt'}, (\overline{s'}Y), (\overline{s'}z)\right], \left[\overline{s'}, \left\{\overline{s'}^{-1}(\overline{s'}z)\right\} \cup (\overline{t}^{-1}W), (\overline{t}^{-1}x)\right]\right\rangle \leftrightarrow \left\langle \left[\overline{rs}, W, x\right], \left[\overline{t}, \left\{(\overline{t}^{-1}x)\right\} \cup Y, z\right]\right\rangle$
and we are back where we started, as required.

XXX patch/patch

XXX conflictor/patch

XXX patch/conflictor

XXX conflictor/conflictor ■

XXX We need to do something about this one. We probably need to add the cases for inverse conflicting patches in the commute rules. This will have the side-effect that we won't need to strengthen the other lemmata.

Lemma 13.3 (catch-commute-square)
$\forall p \in \mathbf{P}, q \in \mathbf{P}, \forall p' \in \mathbf{P}, q' \in \mathbf{P} \cdot$
$(\langle p, q\rangle \leftrightarrow \langle q', p'\rangle) \Leftrightarrow \left(\left\langle q'^{-1}, p\right\rangle \leftrightarrow \left\langle p', q^{-1}\right\rangle\right)$

Explanation
This is Lemma 7.3 restated for catches.

Proof
XXX ■

XXX This needs to be strengthened to only talk about catches adjacent in a repo:

Lemma 13.4 (catch-commute-preserves-commute)
$\forall p \in \mathbf{P}, q \in \mathbf{P}, r \in \mathbf{P}, p' \in \mathbf{P}, q' \in \mathbf{P}, r' \in \mathbf{P} \cdot$
$(\langle p, qr\rangle \leftrightarrow \langle q'r', p'\rangle) \Rightarrow \left(\left(q \underset{?}{\leftrightarrow} r\right) \Leftrightarrow \left(q' \underset{?}{\leftrightarrow} r'\right)\right)$

Explanation
This is Lemma ?? restated for catches.

Proof
XXX ■

XXX This needs to be strengthened to only talk about catches adjacent in a repo:

Lemma 13.5 (catch-commute-consistent)
$\forall p \in \mathbf{P}, q \in \mathbf{P}, r \in \mathbf{P}, p' \in \mathbf{P}, q' \in \mathbf{P}, r' \in \mathbf{P} \cdot$
$(\langle q, r\rangle \leftrightarrow \langle r', q'\rangle) \Rightarrow ((\langle p, qr\rangle \leftrightarrow \langle \_, p'\rangle) \Leftrightarrow (\langle p, r'q'\rangle \leftrightarrow \langle \_, p'\rangle))$

Explanation
This is Lemma ?? restated for catches.

Proof
XXX ∎

```
End catch_patch_universe.
```

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14 Hunks

```
Module Export HUNKS.
```

End-to-end proof for hunks.

```
Require Import Arith.
Require Import Compare_dec.
Require Import Setoid.

Require Import unnamed_patches.
Require Import names.
Require Import named_patches.
Require Import patch_universes.
Require Import invertible_patchlike.

(*  The offset, remove and add values are the number of lines
   the hunk skips over, removes and adds respectively. *)
Inductive Hunk : Set
    := MkHunk : ∀ (offset remove add : nat), Hunk.

Definition hunkInverse (x : Hunk) : Hunk :=
    match x with
    MkHunk xOff xRemove xAdd ⇒ MkHunk xOff xAdd xRemove
    end.
Notation "p ^u" := (hunkInverse p).

(*  This isn't the best definition of hunk commute that we can make,
   but it is a simple and consistent definition *)
Definition hunkCommute (x y y' x' : Hunk) : Prop :=
    match x, y, y', x' with
    MkHunk xOff xRemove xAdd, MkHunk yOff yRemove yAdd,
```

```
    MkHunk yOff' yRemove' yAdd', MkHunk xOff' xRemove' xAdd' ⇒
    (xRemove = xRemove') ∧ (xAdd = xAdd') ∧
    (yRemove = yRemove') ∧ (yAdd = yAdd') ∧
    (
        (
            (xOff + xAdd < yOff) ∧
            (xOff' = xOff) ∧
            (yOff' = yOff - xAdd + xRemove)
        )
        ∨
        (
            (yOff + yRemove < xOff) ∧
            (xOff' = xOff - yRemove + yAdd) ∧
            (yOff' = yOff)
        )
    )
    end.
Notation "« p , q » <~>u « q' , p' »" := (hunkCommute p q q' p').

Definition HunkCommutable : Hunk → Hunk → Prop
 := fun (p : Hunk) ⇒ fun (q : Hunk) ⇒
    (∃ p' : Hunk, ∃ q' : Hunk,
    «p, q» <~>u «q', p'»).
Notation "p <~?~>u q" := (HunkCommutable p q).

Lemma HunkCommuteUnique :
      ∀ (p : Hunk) (q : Hunk)
              (p' : Hunk) (q' : Hunk)
              (p'' : Hunk) (q'' : Hunk),
      «p, q» <~>u «q', p'»
   ∧ «p, q» <~>u «q'', p''»
   → (p' = p'') ∧ (q' = q'').
Proof.
intros.
unfold hunkCommute in H.
destruct p.
destruct q.
destruct p'.
destruct q'.
destruct p''.
destruct q''.
destruct H as [H1 H2].
split; f_equal; omega.
Qed.

Lemma HunkCommutable_dec :
      ∀ (p : Hunk) (q : Hunk),
      {p <~?~>u q} + {~(p <~?~>u q)}.
Proof with auto.
intros.
unfold HunkCommutable.
destruct p as [pOff pRemove pAdd].
destruct q as [qOff qRemove qAdd].
unfold hunkCommute.
```

```
destruct (le_gt_dec qOff (pOff + pAdd)).
    destruct (le_gt_dec pOff (qOff + qRemove)).
        (*  This is the "do not commute" case *)
        right.
        intro.
        destruct H as [p' [q' H]].
        destruct p'.
        destruct q'.
        omega.
    (*  Commute option 2 *)
    left.
    ∃ (MkHunk (pOff - qRemove + qAdd) pRemove pAdd).
    ∃ (MkHunk qOff qRemove qAdd).
    split...
    split...
    split...
(*  Commute option 1 *)
left.
∃ (MkHunk pOff pRemove pAdd).
∃ (MkHunk (qOff - pAdd + pRemove) qRemove qAdd).
split...
split...
split...
Qed.

Lemma HunkCommuteSelfInverseOneWay :
      ∀ (p : Hunk) (q : Hunk)
              (p' : Hunk) (q' : Hunk),
      («p, q» <~>u «q', p'») →
      («q', p'» <~>u «p, q»).
Proof with auto.
intros.
destruct p.
destruct q.
destruct p'.
destruct q'.
unfold hunkCommute in H.
unfold hunkCommute.
omega.
Qed.

Lemma HunkCommuteSelfInverse :
      ∀ (p : Hunk) (q : Hunk)
              (p' : Hunk) (q' : Hunk),
      («p, q» <~>u «q', p'») ↔
      («q', p'» <~>u «p, q»).
Proof.
intros.
split; apply HunkCommuteSelfInverseOneWay.
Qed.

Lemma HunkCommuteSquare :
      ∀ (p : Hunk) (q : Hunk)
              (p' : Hunk) (q' : Hunk),
```

```
      «p, q» <~>u «q', p'» →
      « q'^u, p» <~>u «p', q^u».
Proof.
intros.
destruct p.
destruct q.
destruct p'.
destruct q'.
unfold hunkCommute in H.
unfold hunkCommute.
unfold hunkInverse.
omega.
Qed.

Lemma HunkCommuteConsistent1 :
      ∀ (p1 : Hunk)
            (q1 : Hunk)
            (r1 : Hunk)
            (q2 : Hunk)
            (r2 : Hunk)
            (q3 : Hunk)
            (r3 : Hunk)
            (p3 : Hunk)
            (p5 : Hunk),
      «q1, r1» <~>u «r2, q2»
   → «p1, q1» <~>u «q3, p5»
   → «p5, r1» <~>u «r3, p3»
   → ∃ r4 : Hunk,
      ∃ q4 : Hunk,
      ∃ p6 : Hunk,
      «q3, r3» <~>u «r4, q4» ∧
      «p1, r2» <~>u «r4, p6» ∧
      «p6, q2» <~>u «q4, p3».
Proof with auto.
intros.
destruct p1 as [p1Offset p1Remove p1Add].
destruct q1 as [q1Offset q1Remove q1Add].
destruct r1 as [r1Offset r1Remove r1Add].
destruct q2 as [q2Offset q2Remove q2Add].
destruct r2 as [r2Offset r2Remove r2Add].
destruct p3 as [p3Offset p2Remove p2Add].
destruct q3 as [q3Offset q3Remove q3Add].
destruct r3 as [r3Offset r3Remove r3Add].
destruct p5 as [p5Offset p5Remove p5Add].
unfold hunkCommute in ×.
intuition.

(*  Case 1: p changes before q, q changes before r *)
∃ (MkHunk (r3Offset - q3Add + q3Remove) r3Remove r3Add).
∃ (MkHunk q3Offset q3Remove q3Add).
∃ (MkHunk p1Offset p1Remove p1Add).
subst.
intuition.
```

```
(*  Case 3: q changes before p, p changes before r *)
∃ (MkHunk (r3Offset - q3Add + q3Remove) r3Remove r3Add).
∃ (MkHunk q3Offset q3Remove q3Add).
∃ (MkHunk p1Offset p1Remove p1Add).
subst.
intuition.
(*  Case 4: q changes before r, r changes before p *)
∃ (MkHunk (r3Offset - q3Add + q3Remove) r3Remove r3Add).
∃ (MkHunk q3Offset q3Remove q3Add).
∃ (MkHunk (p1Offset - r1Remove + r1Add) p1Remove p1Add).
subst.
intuition.
(*  Case 5: p changes before r, r changes before q *)
∃ (MkHunk r3Offset r3Remove r3Add).
∃ (MkHunk (q3Offset - r3Remove + r3Add) q3Remove q3Add).
∃ (MkHunk p1Offset p1Remove p1Add).
subst.
intuition.
(*  Case 6: r changes before p, p changes before q *)
∃ (MkHunk r3Offset r3Remove r3Add).
∃ (MkHunk (q3Offset - r3Remove + r3Add) q3Remove q3Add).
∃ (MkHunk (p1Offset - r1Remove + r1Add) p1Remove p1Add).
subst.
intuition.
(*  Case 8: r changes before q, q changes before p *)
∃ (MkHunk r3Offset r3Remove r3Add).
∃ (MkHunk (q3Offset - r3Remove + r3Add) q3Remove q3Add).
∃ (MkHunk (p1Offset - r1Remove + r1Add) p1Remove p1Add).
subst.
intuition.
Qed.

Lemma HunkCommuteConsistent2 :
      ∀ (p3 : Hunk)
              (q3 : Hunk)
              (r3 : Hunk)
              (q4 : Hunk)
              (r4 : Hunk)
              (q1 : Hunk)
              (r1 : Hunk)
              (p1 : Hunk)
              (p5 : Hunk),
      «q3 , r3» <~>u «r4 , q4»
   → «r3 , p3» <~>u «p5 , r1»
   → «q3 , p5» <~>u «p1 , q1»
   → ∃ r2 : Hunk,
      ∃ q2 : Hunk,
      ∃ p6 : Hunk,
      «q1 , r1» <~>u «r2 , q2» ∧
      «q4 , p3» <~>u «p6 , q2» ∧
      «r4 , p6» <~>u «p1 , r2».
Proof with auto.
```

```
intros.
destruct p3 as [p3Offset p3Remove p3Add].
destruct q3 as [q3Offset q3Remove q3Add].
destruct r3 as [r3Offset r3Remove r3Add].
destruct q4 as [q4Offset q4Remove q4Add].
destruct r4 as [r4Offset r4Remove r4Add].
destruct p1 as [p1Offset p1Remove p1Add].
destruct q1 as [q1Offset q1Remove q1Add].
destruct r1 as [r1Offset r1Remove r1Add].
destruct p5 as [p5Offset p5Remove p5Add].
unfold hunkCommute in ×.
intuition.

(*  Case 1: q changes before r, r changes before p *)
∃ (MkHunk (r1Offset - q1Add + q1Remove) r1Remove r1Add).
∃ (MkHunk q1Offset q1Remove q1Add).
∃ (MkHunk (p3Offset - q4Add + q4Remove) p1Remove p1Add).
subst.
intuition.

(*  Case 3: q changes before p, p changes before r *)
∃ (MkHunk (r1Offset - q1Add + q1Remove) r1Remove r1Add).
∃ (MkHunk q1Offset q1Remove q1Add).
∃ (MkHunk (p3Offset - q4Add + q4Remove) p1Remove p1Add).
subst.
intuition.

(*  Case 4: p changes before q, q changes before r *)
∃ (MkHunk (r1Offset - q1Add + q1Remove) r1Remove r1Add).
∃ (MkHunk q1Offset q1Remove q1Add).
∃ (MkHunk p1Offset p1Remove p1Add).
subst.
intuition.

(*  Case 5: r changes before q, q changes before p *)
∃ (MkHunk r1Offset r1Remove r1Add).
∃ (MkHunk (q1Offset - r1Remove + r1Add) q1Remove q1Add).
∃ (MkHunk (p3Offset - q4Add + q4Remove) p1Remove p1Add).
subst.
intuition.

(*  Case 6: r changes before p, p changes before q *)
∃ (MkHunk r1Offset r1Remove r1Add).
∃ (MkHunk (q1Offset - r1Remove + r1Add) q1Remove q1Add).
∃ (MkHunk p3Offset p1Remove p1Add).
subst.
intuition.

(*  Case 8: p changes before r, r changes before q *)
∃ (MkHunk r1Offset r1Remove r1Add).
∃ (MkHunk (q1Offset - r1Remove + r1Add) q1Remove q1Add).
∃ (MkHunk p1Offset p1Remove p1Add).
subst.
intuition.
Qed.

Lemma hunkInverseInverse : ∀ (p : Hunk), (p ^u)^u = p.
```

```
Proof.
intros.
destruct p.
unfold hunkInverse.
auto.
Qed.

Definition HunkUnnamedPatch : UnnamedPatch :=
    mkUnnamedPatch
        Hunk
        hunkInverse
        hunkInverseInverse
        hunkCommute
        HunkCommuteUnique
        HunkCommutable_dec
        HunkCommuteSelfInverse
        HunkCommuteSquare
        HunkCommuteConsistent1
        HunkCommuteConsistent2.

Instance HunkPartPatchUniverse : PartPatchUniverse (NamedPatch HunkUnnamedPatch)
(NamedPatch HunkUnnamedPatch) := NamedPartPatchUniverse HunkUnnamedPatch.
Instance HunkPatchUniverseInv : PatchUniverseInv (NamedPartPatchUniverse HunkUn-
namedPatch) (NamedPartPatchUniverse HunkUnnamedPatch) := NamedPatchUniverseInv
HunkPartPatchUniverse.
Instance HunkPatchUniverse : PatchUniverse (NamedPatchUniverseInv HunkPartPatchUniverse)
:= NamedPatchUniverse HunkPatchUniverseInv.
Instance HunkInvertiblePatchlike : InvertiblePatchlike (NamedPatchUniverse HunkPatchUniver-
seInv) := NamedInvertiblePatchlike HunkPatchUniverse.

Lemma hunkCommuteInSequenceConsistent :
    ∀ {o oq oqr oqrs ou our ours : NameSet}
            {qsContains : SignedNameSet}
            {qs : Sequence HunkPatchUniverse qsContains o oq}
            {r : NamedPatch HunkUnnamedPatch oq oqr}
            {s : NamedPatch HunkUnnamedPatch oqr oqrs}
            (*  ts omitted for now *)
            {usContains vsContains : SignedNameSet}
            {us : Sequence HunkPatchUniverse usContains o ou}
            {r' : NamedPatch HunkUnnamedPatch ou our}
            {s' : NamedPatch HunkUnnamedPatch our ours}
            {vs : Sequence HunkPatchUniverse vsContains ours oqrs}

            {NilContains s_NilContains r_s_NilContains : SignedNameSet}
            {s'_vsContains r'_s'_vsContains : SignedNameSet}
            {contains : SignedNameSet}

            (nilOK : NilOK oqrs oqrs NilContains)
            (s_NilConsOK : ConsOK oqr s_NilContains NilContains s)
            (r_s_NilConsOK : ConsOK oq r_s_NilContains s_NilContains r)
            (qs_r_s_NilAppendOK : appendOK qs (r :> s :> []) contains)

            (s'_vsConsOK : ConsOK our s'_vsContains vsContains s')
            (r'_s'_vsConsOK : ConsOK ou r'_s'_vsContains s'_vsContains r')
```

```
            (us_r'_s'_vsAppendOK : appendOK us (r' :> (s' :> vs)) contains),
    («qs :+> r :> s :> []» <~~>* «us :+> r' :> s' :> vs»)
 → (pu_nameOf r = pu_nameOf r')
 → (pu_nameOf s = pu_nameOf s')
 → (r <~?~> s)
 → (r' <~?~> s').
Proof.
apply (@commuteInSequenceConsistent
            (NamedPatch HunkUnnamedPatch)
            HunkPartPatchUniverse
            HunkPatchUniverseInv
            HunkPatchUniverse).
Qed.

End HUNKS.
```

A More catch bits

XXX This whole section wants to be merged back in as appropriate.

XXX Merge bits

Conjecture A.1 (merge-commute-cannot-fail)
The commute in the merge definition cannot fail.

Explanation
If it fails then something has gone badly wrong, as merge is not allowed to fail!

Conjecture A.2 (merge-makes-repos)
If $(\overline{c}d) \in \mathbf{R}, (\overline{c}e) \in \mathbf{R}, d' \in \mathbf{C}, e' \in \mathbf{C}, d + e = \langle e', d' \rangle$ then $\overline{c}de' \in \mathbf{R} \wedge \overline{c}ed' \in \mathbf{R}$.

Explanation
This states that merging two valid repositories results in another valid repository.

Conjecture A.3 (merge-effect)
XXX This is rubbish. Fix it. Or just rely on the spec of the effect of a repo, which we haven't given yet.

If $(\overline{c}d) \in \mathbf{R}$, $(\overline{c}e) \in \mathbf{R}$ and $d + e = \langle e', d' \rangle$ then $\mathscr{E}(de') = \mathscr{E}(ed')$ if $\langle d^{-1}, e \rangle \leftrightarrow \langle \_, \_ \rangle$, and id otherwise.

Explanation
XXX Actually, I think this is wrong if the two catches are not both patches.
Anyway, the idea is that we should apply the affects if the merge succeeds, but there should be no net effect if it doesn't.

Conjecture A.4 (merge-symmetric)
If $(\overline{c}d) \in \mathbf{R}$, $(\overline{c}e) \in \mathbf{R}$ and $d + e = \langle e', d' \rangle$ then $e + d = \langle d', e' \rangle$.

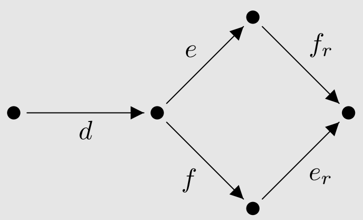
Explanation
Merging d and e is the same as merging e and d.

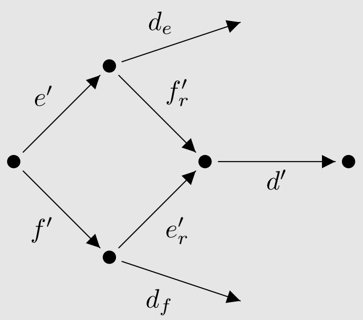
Conjecture A.5 (merge-commute)
If $(\overline{c}d) \in \mathbf{R}$, $(\overline{c}e) \in \mathbf{R}$ and $d + e = \langle e', d' \rangle$ then $\langle d, e' \rangle \leftrightarrow \langle e, d' \rangle$.

Explanation
The two ways of constructing the result of a merge are equivalent.

Conjecture A.6 (merge-commute2)
If $(\bar{c}de) \in \mathbf{R}, (\bar{c}df) \in \mathbf{R}, e + f = \langle f_r, e_r \rangle, \langle d, e \rangle \leftrightarrow \langle e', d_e \rangle, \langle d, f \rangle \leftrightarrow \langle f', d_f \rangle$ then
$\langle d_e, f_r \rangle \leftrightarrow \langle \_, d' \rangle, \langle d_f, e_r \rangle \leftrightarrow \langle \_, d' \rangle, e' + f' = \langle f'_r, e'_r \rangle, d_e + f'_r = \langle \_, d' \rangle, d_f + e'_r = \langle \_, d' \rangle$

Explanation
First, a couple of diagrams may help make it clearer what is going on:





This conjecture says that if d commutes past e and f then we can commute it before or after merging, and get consistent results either way.

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

XXX commute bits

Definition A.1 (catch-commute)
We extend $\leftrightarrow$ to operate on catches, as defined below.

We can break the rules down into 3 classes: Those where the patches are unrelated, and simply commute freely; those where the patches are the result of a conflicting merge, and the conflicts gets reshuffled when they commute; and a fail case for anything else.

Currently we don't give the commute rules for anything involving inverse conflictors. We don't believe that those rules will raise any new problems, though.

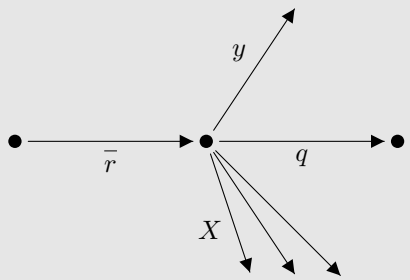
Unrelated

First the simple non-conflicted patch case:
$\langle [p], [q] \rangle \leftrightarrow \langle [q'], [p'] \rangle$ if $\langle p, q \rangle \leftrightarrow \langle q', p' \rangle$

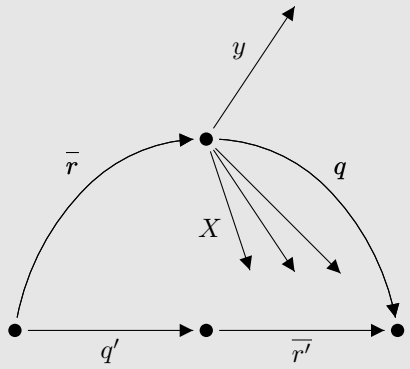
Explanation
If our catches just wrap up patches, then they commute like the underlying patches.

Now commute a conflictor past a patch, where everything goes smoothly:
$\left\langle \left[\overline{r}, X, y\right], [q]\right\rangle \leftrightarrow \left\langle [q'], \left[\overline{r'}, X', y'\right]\right\rangle$ if $\left\langle \overline{r}, q\right\rangle \leftrightarrow \left\langle q', \overline{r'}\right\rangle$
$\left\langle q^{-1}, y\right\rangle \to \langle y'\rangle$
$\left\langle q^{-1}, X\right\rangle \to \langle X'\rangle$

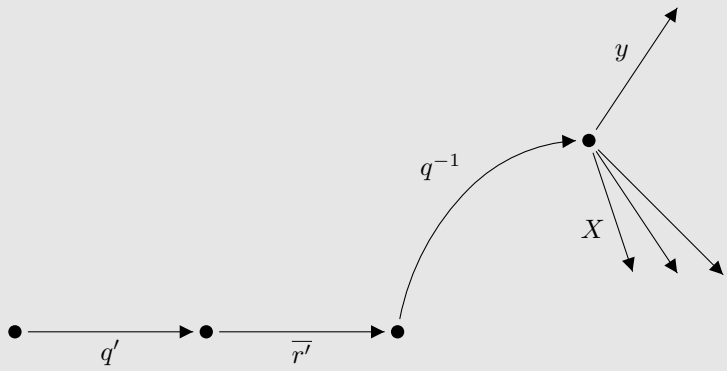
Explanation
OK, now things start to look a little more daunting, but if you work through it then it's really not that bad. We start off with this situation:



Now, we want to commute the two patches, so clearly we need to commute $\overline{r}$ and q, which leaves us here:



The $q'\overline{r'}$ is what we want, but the y and X that should be part of the conflictor have become detached from it. Let's rearrange the diagram a bit to make it clearer what we need to do:



So we need to add q^{-1} into the context of y and X. We have two ways of doing this: We can simply add q^{-1} to the context, which cannot fail (by definition), or we can commute q^{-1} past y and X, which can. Being able to commute patches is good, so if possible we would

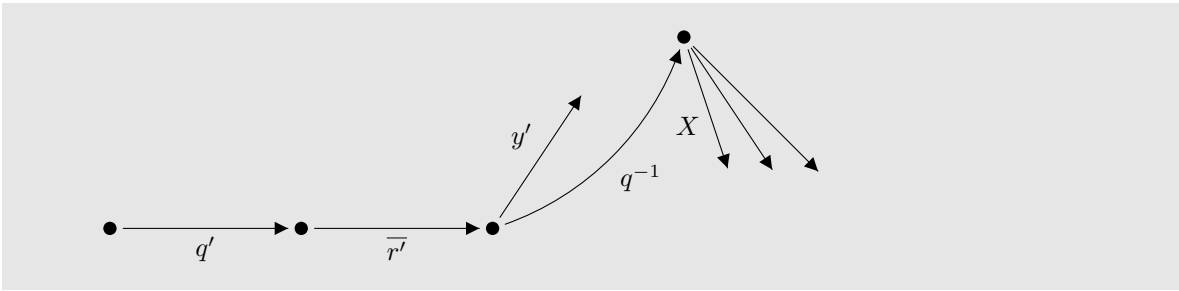
prefer to use the option that doesn't fail; but is that possible?

First, let's consider y, and for the purpose of contradiction, let us assume that the catch commute can suceed even if q^{-1} cannot be commuted past y.

Suppose we have two repos $[s]$ and $[t]$, where s and t conflict. We merge them, giving us $[s]\left[s^{-1}, \{: s\}, : t\right]$, and then record another patch $[u]$, giving us $[s]\left[s^{-1}, \{: s\}, : t\right][u]$. We choose u such that $\left\langle s^{-1}, u\right\rangle \leftrightarrow \left\langle u', s'^{-1}\right\rangle$ and $\left\langle t^{-1}, u\right\rangle \leftrightarrow$ fail. Then we can commute $[u]$ past both catches, giving us $[u][s']\left[s'^{-1}, \{: s'\}, u^{-1} : t\right]$. So far so good.

But before we commute u past, we can first commute s and t, giving us $[t]\left[t^{-1}, \{: t\}, : s\right][u]$. But now it is not possible to commute u past!

This isn't what we want, so we now know that we must require that the commute past y succeeds. This explains why we require $\left\langle q^{-1}, y\right\rangle \to \langle y'\rangle$, and gets us to here:



Now we need to work out whether we need to require that q^{-1} commutes past X or not. Again, for the purpose of contradiction, assume that we don't require this.

Suppose we have three repos $[s]$, $[t]$ and $[u]$, where s conflicts with t and u. Then we merge these three repos to give us something like

$$[s]\left[s^{-1}, \{: s\}, : t\right][\epsilon, \{: s\}, : u]$$

Next record another patch v, which commutes with everything except for s^{-1}. So now we have

$$[s]\left[s^{-1}, \{: s\}, : t\right][\epsilon, \{: s\}, : u][v]$$

Now, we can commute v past the u conflictor, but not the t conflictor (as it cannot commute past the effect, s^{-1}):

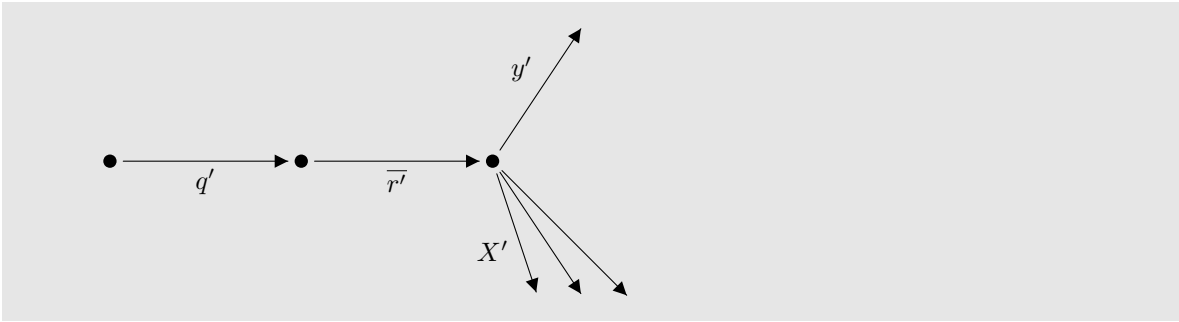
$$[s]\left[s^{-1}, \{: s\}, : t\right][v]\left[\epsilon, \left\{v^{-1} : s\right\}, : u'\right]$$

Likewise, we can first commute the t and u conflictors and then commute v past the t conflictor, but not the u conflictor:

$$[s]\left[s^{-1}, \{: s\}, : u\right][v]\left[\epsilon, \left\{v^{-1} : s\right\}, : t'\right]$$

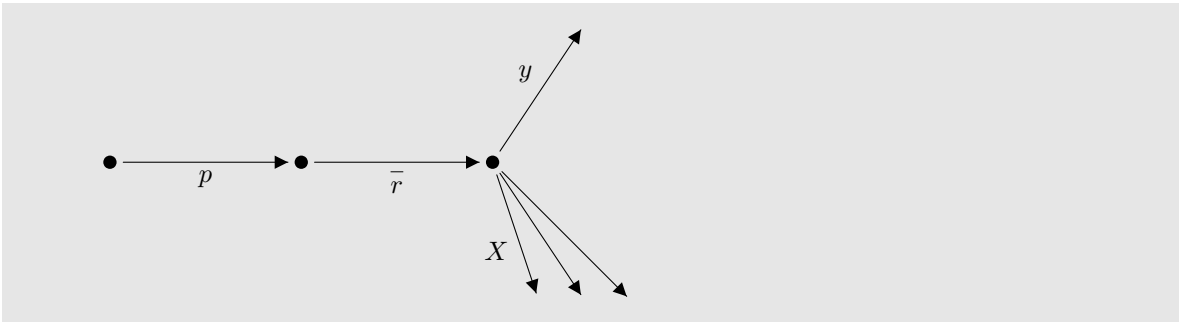
By unpulling patches from these last two repos, we get repos with the patches $\{s, t, v\}$ and $\{s, u, v\}$, but it is not possible to make a repo containing the patches $\{s, v\}$. This causes problems when we try to merge these two repos, as explained in Appendix B, so this is also not something that we want to allow.

This explains why we require $\left\langle q^{-1}, X\right\rangle \to \langle X'\rangle$, and brings us to our destination:

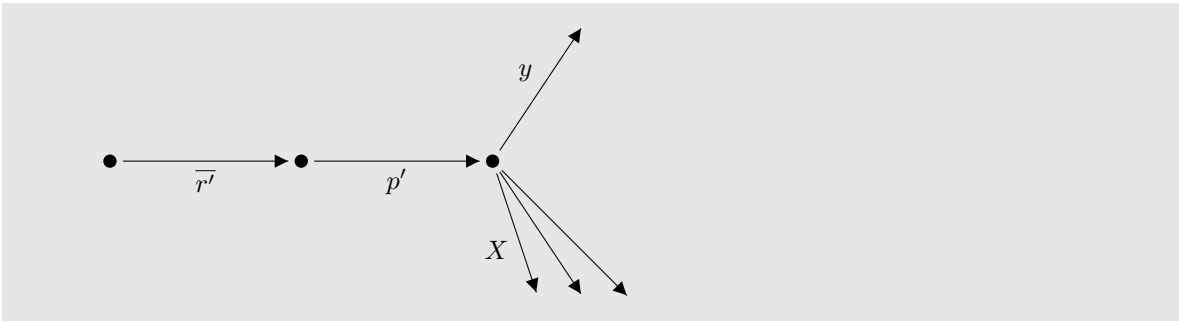


Next we have the case where the patch and conflictor start off the other way round:
$\langle [p], [\overline{r}, X, y] \rangle \leftrightarrow \langle [\overline{r'}, X', y'], [p'] \rangle$ if $\langle p, \overline{r} \rangle \leftrightarrow \langle \overline{r'}, p' \rangle$
$\langle p', X \rangle \to \langle X' \rangle$
$\langle p', y \rangle \to \langle y' \rangle$

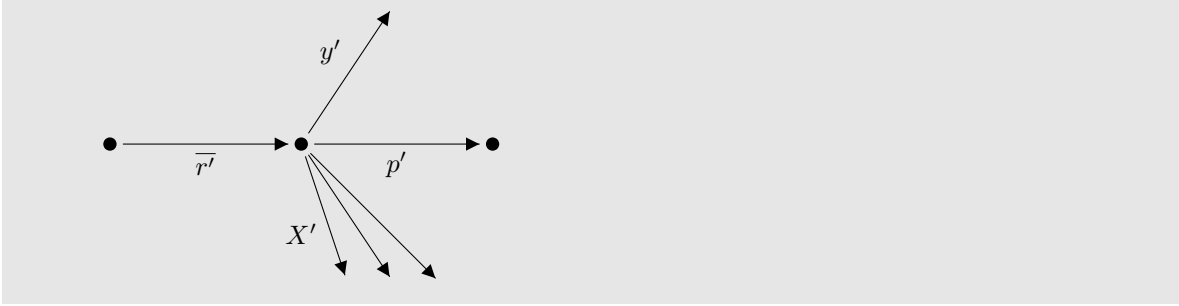
Explanation
This is similar to the previous case. We start off here:



As before, we start off by commuting p and $\overline{r}$:

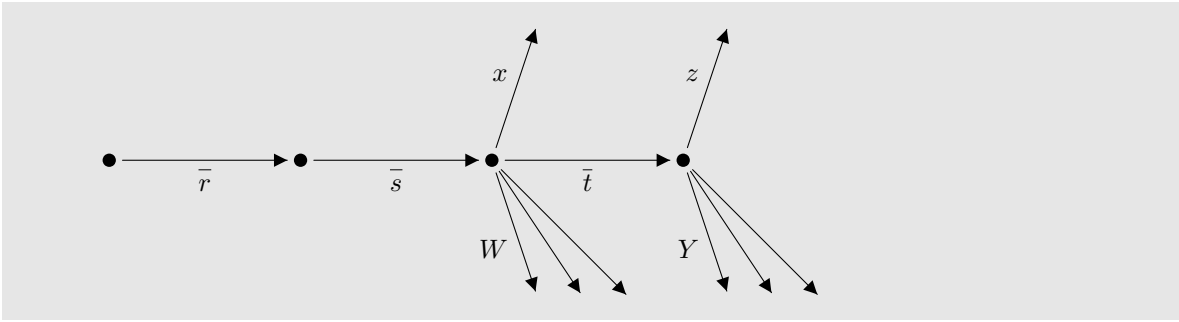


We now need to get X and y into the right context, and as we want commute to be self-inverting, we'd better require that the commute pasts succeed, as we did in the previous case:



And the last and most complex of the unrelated cases, commuting a conflictor past another conflictor:
$\left\langle \left[\overline{rs}, W, x\right], \left[\overline{t}, Y, z\right]\right\rangle \leftrightarrow \left\langle \left[\overline{rt'}, \overline{s'}Y, z'\right], \left[\overline{s'}, \overline{t}^{-1}W, x'\right]\right\rangle$
if $N\left(\overline{r}^{-1}\right) \subseteq N\left(Y\right)$
$N\left(\overline{s}^{-1}\right) \cap N\left(Y\right) = \emptyset$
$\left\langle \overline{s}, \overline{t}\right\rangle \leftrightarrow \left\langle \overline{t'}, \overline{s'}\right\rangle$
$x \leftrightarrows \overline{t}z$
$\forall w \in W \cdot (w \leftrightarrows \overline{t}z)$
$\forall y \in Y \cdot (x \leftrightarrows \overline{t}y)$
$\left\langle \overline{t}^{-1}, x\right\rangle \to \left\langle x'\right\rangle$
$\left\langle \overline{s'}, z\right\rangle \to \left\langle z'\right\rangle$

Explanation
The number of rules and side conditions grows, but if we take it one step at a time we can get through this! We start here:



The first thing to note is that we have $\overline{rs}$ where you probably expected $\overline{t}$. What we have done here is to break up the effect of the first conflictor into those patches that both *conflictors conflict with, $\overline{r}$, and those patches that only the first conflictor conflicts with, $\overline{s}$. $\overline{r}$ won't move; it'll become part of the z conflictor after the commute, thus maintaining the invariant that the first conflictor that conflicts with a patch reverts it. The first two side conditions just say that we have correctly split up the effect of the first conflictor.*

With that out of the way, let's start to look at how we perform the commute. We start, as usual, by commuting the effects of the two patches. Now comes the difficult part: Where do we need to check for conflicts?

First let's consider the repo $[p]\,[q]$, where q depends on p, and the repo $[u]$, where p conflicts with u. If we merge these as $[p]\,[q]\,\left[q^{-1}p^{-1}, \{: p, p : q\}, : u\right]$ then we cannot commute the p and q catches. Therefore, in the alternate merge $[u]\,\left[u^{-1}, \{: u\}, : p\right]\,[, \{: u\}, p : q]$ the catches shouldn't commute either. Therefore we need to check that p and $p : q$ don't conflict, or in general, $x \leftrightarrows \overline{t}z$.

Now let us consider whether we need to worry about z and W conflicting. Suppose first we record o, p and q, which conflict, in separate repos, and then merge them, giving us

$$[o]\,\left[o^{-1}, \{: o\}, : p\right]\,[, \{: o, : p\}, : q]$$

Next record u (which commutes with o, p and q) and v (which commutes with q but not o or p) in two copies of this repo, and merge, giving us

$$[o]\,\left[o^{-1}, \{: o\}, : p\right]\,[, \{: o, : p\}, : q]\,[u]\,\left[u^{-1}, \{: u\}, : v\right]$$

Now, we can commute u and v, giving us

$$[o]\,\left[o^{-1}, \{: o\}, : p\right]\,[, \{: o, : p\}, : q]\,[v]\,\left[v^{-1}, \{: v\}, : u\right]$$

and by the earlier rules we know that the q catch does not commute with the v catch. But we could also have commuted q and u, giving us

$$[o]\left[o^{-1},\{:o\},:p\right][u]\left[,\{:o',:p'\},:q'\right]\left[u^{-1},\{:u\},:v\right]$$

Now we musn't be allowed to commute the q and v conflictors, and we must therefore, in the general case, check that z and W do not conflict. This gives us the $\forall w \in W \cdot (w \leftrightarrows \overline{t}z)$ side condition, and by considering commuting the patches back the other way we must also have $\forall y \in Y \cdot (x \leftrightarrows \overline{t}y)$.

If this is satisfied then we know that the commute pasts will succeed.

This only leaves conflicts between W and Y. However, I claim that it is not possible to get into a situation where there are any conflicts here; Any patch in Y would have had to be commuted past the x conflictor at some point in order to get into this situation, and therefore cannot conflict with W.

Conflicted merge

Now we get to the interesting cases. First, if we have a patch, and a conflictor that has only conflicted with that patch, then they swap places:
$\left\langle [p], \left[p^{-1},\{:p\},:q\right]\right\rangle \leftrightarrow \left\langle [q], \left[q^{-1},\{:q\},:p\right]\right\rangle$

Explanation
This should make sense if you consider that the result of merging two conflicting patches p and q is $[p]\left[p^{-1},\{:p\},:q\right]$ (i.e., from right to left, the second catch represents q, conflicts with p, and as it is the first patch to conflict with p it has to invert p's effect), or equivalently $[q]\left[q^{-1},\{:p\},:q\right]$.

If we have two conflictors, but the one on the right only conflicts with the one on the left, then it becomes a patch after it has commuted:
$\left\langle \left[\overline{r},X,y\right], \left[\epsilon,\{y\},\overline{r}^{-1}:q\right]\right\rangle \leftrightarrow \left\langle [q], \left[q^{-1}\overline{r},\left\{\overline{r}^{-1}:q\right\}\cup X,y\right]\right\rangle$

Explanation
This case comes up when you merge a patch q with a conflictor representing p. If q is the first in the resulting sequence then it is still just $[q]$, and the q conflictor must record that it conflicts with p. On the other hand, if p comes after p then it must also turn into a conflictor, as it needs to record that is has conflicted with p.

And the inverse of the previous case:
$\left\langle [p], \left[p^{-1}\overline{r},\left\{\overline{r}^{-1}:p\right\}\cup X,y\right]\right\rangle \leftrightarrow \left\langle \left[\overline{r},X,y\right], \left[\epsilon,\{y\},\overline{r}^{-1}:p\right]\right\rangle$

Explanation
This is just the inverse of the previous case.

And now the case where both are conflictors, and conflict with each other:
$\left\langle \left[\overline{rs},W,x\right], \left[\overline{t},\left\{\overline{t}^{-1}x\right\}\cup Y,z\right]\right\rangle \leftrightarrow \left\langle \left[\overline{rt'},\overline{s'}Y,\overline{s'}z\right], \left[\overline{s'},\{z\}\cup\overline{t}^{-1}W,\overline{t}^{-1}x\right]\right\rangle$
if $\left\langle \overline{s},\overline{t}\right\rangle \leftrightarrow \left\langle \overline{t'},\overline{s'}\right\rangle$
$N\left(\overline{r}^{-1}\right) \subseteq N(Y)$
$N\left(\overline{s}^{-1}\right) \cap N(Y) = \emptyset$

Explanation
In this case we just move the conflict from one to the other. The splitting of the effect of the z conflictor into two parts is the same as we saw earlier, in the "unrelated" conflict-conflictor commute.

Fail

Finally, if none of the above hold, then
$\langle c, d\rangle \hookrightarrow$ fail

Conjecture A.7 (catch-commute-conflicting-patches-succeeds)
$\forall(\overline{c}de) \in \mathbf{R}\cdot$
$d \in \mathscr{C}(e) \Rightarrow (\langle d, e\rangle \hookrightarrow \langle \_, \_\rangle)$

> **Explanation**
> *If two patches conflict (and thus, were merged at some point in the past), then they can be commuted.*

Conjecture A.8 (catch-commute-unique)
$\forall(\overline{c}de) \in \mathbf{R}, r \in (\mathbf{C} \times \mathbf{C}) \cup \{\text{fail}\}, s \in (\mathbf{C} \times \mathbf{C}) \cup \{\text{fail}\}\cdot$
$(\langle d, e\rangle \hookrightarrow r) \wedge (\langle d, e\rangle \hookrightarrow s) \Rightarrow r = s$

> **Explanation**
> *XXX*

Conjecture A.9 (catch-commute-valid)
$\forall(\overline{c}de) \in \mathbf{R}, d' \in \mathbf{C}, e' \in \mathbf{C}\cdot$
$(\langle d, e\rangle \hookrightarrow \langle e', d'\rangle) \Rightarrow (\overline{c}e'd') \in \mathbf{R}$

> **Explanation**
> *If we have a valid repository (one in which the invariants are all satisfied) and we commute two catches, then the result is also a valid repository.*

Conjecture A.10 (catch-commute-self-inverse)
$\forall(\overline{c}de) \in \mathbf{R}, \forall d' \in \mathbf{C}, e' \in \mathbf{C}\cdot$
$(\langle d, e\rangle \hookrightarrow \langle e', d'\rangle) \Leftrightarrow (\langle e', d'\rangle \hookrightarrow \langle d, e\rangle)$

> **Explanation**
> *XXX*

Conjecture A.11 (catch-commute-square)
$\forall(\overline{c}d) \in \mathbf{R}, (\overline{c}e) \in \mathbf{R}, \forall d' \in \mathbf{C}, e' \in \mathbf{C}\cdot$
$(\langle d^{-1}, e\rangle \hookrightarrow \langle e', d'^{-1}\rangle) \Leftrightarrow (\langle e^{-1}, d\rangle \hookrightarrow \langle d', e'^{-1}\rangle)$

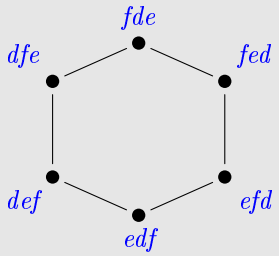
> **Explanation**
> *XXX*

Conjecture A.12 (catch-commute-preserves-effect)
$\forall(\overline{c}de) \in \mathbf{R}, \forall d' \in \mathbf{C}, e' \in \mathbf{C}\cdot$
$(\langle d, e\rangle \hookrightarrow \langle e', d'\rangle) \Rightarrow \mathscr{E}(de) = \mathscr{E}(e'd')$

> **Explanation**
> *XXX*

Conjecture A.13 (catch-commute-associates)
$\forall(\overline{c}def) \in \mathbf{R}, d_e \in \mathbf{C}, d_f \in \mathbf{C}, d_{ef} \in \mathbf{C}, e_d \in \mathbf{C}, e_f \in \mathbf{C}, e_{df} \in \mathbf{C}, f_d \in \mathbf{C}, f_e \in \mathbf{C}, f_{de} \in \mathbf{C}\cdot$
$(\langle e, f\rangle \hookrightarrow \langle f_e, e_f\rangle) \wedge (\langle d, e\rangle \hookrightarrow \langle e_d, d_e\rangle) \wedge (\langle d_e, f\rangle \hookrightarrow \langle f_d, d_{ef}\rangle)$
$\Leftrightarrow (\langle e_d, f_d\rangle \hookrightarrow \langle f_{de}, e_{df}\rangle) \wedge (\langle e_{df}, d_{ef}\rangle \hookrightarrow \langle d_f, e_f\rangle) \wedge (\langle f_{de}, d_f\rangle \hookrightarrow \langle d, f_e\rangle)$

Explanation
XXX i.e. that commute associates, holds for catches too. Here's the diagram again for convenience:



B Named Patch Motivation

When you record a "patch" in camp, you create a *mega patch* composed of many catches (which, at the point at which you record it, are all just patches). You need to give a name to this mega patch, but to avoid confusion with the names given to patches and catches we'll say that mega patches have a *title*.

A non-obvious result is that, if we don't give names to patches, then dependencies of mega patches *cannot* behave as one would expect when duplicate changes are involved. Furthermore, one can construct situations where the simple, natural merge algorithm fails.

The detail of what is inside a conflictor is irrelevant, as this is a universal property, so for this section we will simply write $[\overline{p}, q]$ for a conflictor representing q that has effect $\overline{p}$.

To start off with, we record a mega patch p in one repo, and q in another repo. p and q contain the same single patch. We also record a mega patch r that depends on p, i.e. p and r do not commute. So we have three repos: p, q, pr.

Next merge p and q, resulting in $p\,[p^{-1}, q]$, and then merge this with pr, resulting in $p\,[p^{-1}, q]\,[, r]$.

Now, this must commute to $pr\,[r^{-1}p^{-1}, q]$, and we can then unpull the q conflictor to get pr.

But going back to $p\,[p^{-1}, q]\,[, r]$, this must also commute to $q\,[q^{-1}, p]\,[, r]$. But if the patches in p and q are not named, and are identical to each other, then $[p^{-1}, q]$ and $[q^{-1}, p]$ look identical to the r conflictor! So this must commute to $qr\,[r^{-1}q^{-1}, p]$ and again we can unpull to get qr. But in the land of named patches, r is supposed to depend on p!

This is disturbing enough, but now suppose that we create these pr and qr repos and then try to merge them. We first want to get all the mega patches that are common to both repos out of the way. We look at the titles of the mega patches in each repo and conclude that r is common to both. We thus want to commute the repos so that they are $r'p'$ and $r'q'$ respectively. But r depends on p/q, so this commute fails!

XXX Change the /

XXX Say key point is merge equates by name, commute by content